MULTI SCALE MODELING OF FIBER AND FABRIC REINFORCED CEMENT BASED COMPOSITES

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ABSTRACT

With an increased use of fiber reinforced concrete in structural applications, proper characterization techniques and development of design guides are needed. This dissertation presents a multi-scale modeling approach for fiber and fabric reinforced cement-based composites. A micromechanics-based model of the yarn pullout mechanism due to the failure of the interfacial zone is presented. The effect of mechanical anchorage of transverse yarns is simulated using nonlinear spring elements. The yarn pullout mechanism was used in a meso-scale modeling approach to simulate the yarn bridging force in the crack evolution process. The tensile stress-strain response of a tension specimen that experiences distributed cracking can be simulated using a generalized finite difference approach. The stiffness degradation, tension stiffening, crack spacing evolution, and crack width characteristics of cement composites can be derived using matrix, interface and fiber properties.

The theoretical models developed for fabric reinforced cement composites were then extended to cover other types of fiber reinforced concrete such as shotcrete, glass fiber reinforced concrete (GFRC), steel fiber reinforced concrete (SFRC), ferrocement and other conventional composite systems. The uniaxial tensile stress-strain response was used to formulate a generalized parametric closed-form solution for predicting flexural behavior of various composites at the macro-structural level. The flexural behaviors of these composites were modeled in a unified manner by means of a moment-curvature relationship based on the uniaxial material models.

A variety of theoretical models were developed to address the various mechanisms including: an analytical yarn pullout model; a nonlinear finite difference fabric pullout
model; a nonlinear finite difference tension model; closed-form solutions for strain-
softening materials; closed-form solutions for strain-softening/hardening materials; and
closed-form solutions for hybrid reinforced concrete models. These theoretical models
were verified by independent experimental results obtained from various sources in
literature. In addition, the closed-form solutions for flexural modeling were simplified to
obtain design equations. Two guidelines were proposed for the design of flexural
members using strain-softening fiber reinforced concrete with and without conventional
steel rebar, and using strain-hardening glass fiber reinforced concrete. The proposed
methodologies enable one to conduct experiments to obtain material parameters and
design structural members using a unified and cohesive formulation.
DEDICATION

This dissertation is dedicated to my loving family for their support, patience and understanding during the last four years.
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# TABLE OF CONTENTS

LIST OF TABLES .......................................................................................................................... xv

LIST OF FIGURES ...................................................................................................................... xviii

1. INTRODUCTION .................................................................................................................. 1

1.1. Overview .......................................................................................................................... 1

1.2. Fabric Reinforced Cement Composites ......................................................................... 5

1.2.1. Fibers and Fabrics ...................................................................................................... 5

1.2.2. Concrete Matrix ......................................................................................................... 6

1.2.3. Manufacturing Methods ............................................................................................ 7

1.2.4. Commercial Products ............................................................................................... 8

1.3. Objectives ....................................................................................................................... 9

1.3.1. Micromechanics Modeling of Fabric-Cement Matrix Interface ............................ 10

1.3.2. Mesomachanics Modeling of Crack Evolution in Tension Specimen .................. 11

1.3.3. Macromachanics Modeling of Structural Applications ........................................ 12

1.3.4. Fiber Reinforced Concrete with Conventional Steel Rebar ................................. 13

1.4. Conclusions ..................................................................................................................... 14

2. LITERATURE REVIEW ..................................................................................................... 30

2.1. Interface Properties between Fiber and Matrix ............................................................ 30

2.1.1. Perfect Interface Model (Stress Approach) ............................................................... 30

2.1.2. Fracture Mechanical Model (Energy Approach) ....................................................... 31

2.1.3. Cohesive Interface Model (Stress Approach) .......................................................... 32
<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1.4. Approach for Fabric Reinforced Pullout Model</td>
<td>34</td>
</tr>
<tr>
<td>2.1.5. Fabric Pullout Model</td>
<td>35</td>
</tr>
<tr>
<td>2.2. Tension Responses of Fiber Reinforced Cement Composites</td>
<td>36</td>
</tr>
<tr>
<td>2.2.1 Empirical Approach</td>
<td>37</td>
</tr>
<tr>
<td>2.2.2 Analytical Approach</td>
<td>39</td>
</tr>
<tr>
<td>2.2.3 Numerical Approach</td>
<td>41</td>
</tr>
<tr>
<td>2.3. Flexural Modeling of Fiber Reinforced Cement Composites</td>
<td>44</td>
</tr>
<tr>
<td>2.4. Design Guides for Fiber Reinforced Concrete</td>
<td>50</td>
</tr>
<tr>
<td>2.5. The Relationship between Stress Strain and Stress Crack Width Model</td>
<td>55</td>
</tr>
<tr>
<td>3. MODELING OF FIBER-MATRIX INTERFACES</td>
<td>71</td>
</tr>
<tr>
<td>3.1. Introduction</td>
<td>71</td>
</tr>
<tr>
<td>3.2. Analytical Fabric Pullout Model</td>
<td>74</td>
</tr>
<tr>
<td>3.2.1. Mathematical Derivations</td>
<td>75</td>
</tr>
<tr>
<td>3.2.1.1. Pullout Response in Elastic Stages (I)</td>
<td>76</td>
</tr>
<tr>
<td>3.2.1.2. Pullout Response in Nonlinear Stage (II)</td>
<td>77</td>
</tr>
<tr>
<td>3.2.1.3. Pullout Response in Dynamic Stage (III)</td>
<td>78</td>
</tr>
<tr>
<td>3.2.2. Algorithms for Pullout Simulations</td>
<td>79</td>
</tr>
<tr>
<td>3.2.3. Pullout Experiments</td>
<td>80</td>
</tr>
<tr>
<td>3.2.3.1. Mix Design</td>
<td>80</td>
</tr>
<tr>
<td>3.2.3.2. Fabric Types</td>
<td>81</td>
</tr>
<tr>
<td>3.2.3.3. Sample Preparation</td>
<td>82</td>
</tr>
</tbody>
</table>
3.2.3.4. Pullout Tests .................................................................................. 83
3.2.4. Simulations of Pullout Tests .............................................................. 85
3.2.5. Results and Discussions ................................................................. 87
3.3. Numerical Fabric Pullout Model ......................................................... 89
  3.3.1. Free Form Nonlinear Material Models .......................................... 90
  3.3.2. Derivation of Finite Difference Fabric Pullout Model .................... 91
    3.3.2.1. Finite Difference Equilibrium Equations .................................. 91
    3.3.2.1.1. Boundary Condition at the Left End ................................... 92
    3.3.2.1.2. Interior Nodes .................................................................. 93
    3.3.2.1.3. Boundary Condition at the Right End ............................... 94
    3.3.2.2. Matrix Assemblage ................................................................. 95
    3.3.2.3. Material Stiffness Updating and Convergence ......................... 96
  3.3.3. Parametric Studies of Material Parameters ................................. 97
    3.3.3.1. Effect of Slack and Cross Yarn Junctions to the Pullout
    Responses ....................................................................................... 98
    3.3.3.2. Effect of Bond Strength, Degree of Slack and Transverse
    Yarn Strength ................................................................................ 99
  3.3.4. Comparison with Other Fiber Pullout Models ............................. 100
  3.3.5. Case Study of AR-Glass Fabric Reinforced Cement .................... 101
    3.3.5.1. Numerical Investigation by Fabric Pullout Model ................... 102
  3.3.6. Results and Discussions ............................................................. 103
4. TENSILE MODELING OF FIBER REINFORCED CEMENT COMPOSITES

4.1. Introduction

4.2. Tension Stiffening in Fabric Reinforced Cement Composites

4.2.1. Material Constitutive Behaviors and Cracking Criterion

4.2.2. Finite Difference Tension Stiffening Model

4.2.2.1. Finite Difference Equilibrium Equations

4.2.2.1.1. Boundary at the Left End

4.2.2.1.2. Interior Nodes

4.2.2.1.3. Boundary Condition at the Crack Face

4.2.2.1.4. Boundary Condition at the Right End Node of the Last Segment

4.2.2.2. Matrix Assemblage

4.2.2.3. Material Stiffness Updating and Convergence Check

4.2.3. Algorithm for Simulation of Tensile Responses under Load Control Test

4.2.4. Parametric Studies of Material Parameters

4.2.4.1. Mechanical Responses of a Numerical Base Model

4.2.4.2. The Effect of Four Main Factors

4.2.5. Simulations of Tension Members

4.2.5.1. Simulation of Reinforced Concrete Tension Members
4.2.5.2. Simulation of AR-Glass Fabric Reinforced Concrete with Equivalent Bond Slip Model .............................................................. 171

4.2.5.3. Simulation of AR-Glass Fabric Reinforced Concrete with Bond-Slip and Spring Model ............................................................... 173

4.4. Chapter Conclusions ......................................................................................................................................................... 175

5. FLEXURAL MODELING OF FIBER REINFORCED CEMENT COMPOSITES ........................................................................................................... 207

5.1. Introduction ........................................................................................................................................................................................ 207

5.2. Strain-Softening Fiber Reinforced Concrete ............................................................................................................................... 210

5.2.1. Material Models ......................................................................................................................................................................... 210

5.2.2. Derivation of Moment-Curvature Response ............................................................................................................................ 211

5.2.3. Moment-Curvature Response at Limit State ............................................................................................................................. 214

5.2.4. Simplified Moment-Curvature Response ................................................................................................................................. 215

5.2.5. Simplified Closed-Form Solutions for Load-Deflection Responses ................................................................. 218

5.2.6. Algorithm to Predict Load-Deflection Responses .................................................................................................................. 220

5.2.7. Parametric Studies of Material Parameters .......................................................................................................................... 222

5.2.8. Simulations of Strain-Softening Steel Fiber Reinforced Concrete ..................................................................................... 223

5.2.9. Discussions ................................................................................................................................................................................... 225

5.3. Extended Model for Strain-Softening and Strain-Hardening ................................................................................................. 226

5.3.1. Material Models ......................................................................................................................................................................... 226

5.3.2. Derivation of Moment-Curvature Response ............................................................................................................................ 229
<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3.3. Crack Localization Rules</td>
<td>231</td>
</tr>
<tr>
<td>5.3.4. Algorithm to Predict Load-Deflection Responses</td>
<td>233</td>
</tr>
<tr>
<td>5.3.5. Parametric Studies of Material Parameters</td>
<td>234</td>
</tr>
<tr>
<td>5.3.6. Simulations of Strain-Softening and Hardening Fiber Reinforced Concrete</td>
<td>236</td>
</tr>
<tr>
<td>5.3.6.1. Simulation of Steel Fiber Reinforced Concrete (SFRC)</td>
<td>236</td>
</tr>
<tr>
<td>5.3.6.2. Simulation of Engineered Cementitious Composite (ECC)</td>
<td>239</td>
</tr>
<tr>
<td>5.3.7. Discussions</td>
<td>240</td>
</tr>
<tr>
<td>5.4. Chapter Conclusions</td>
<td>240</td>
</tr>
<tr>
<td>6. FLEXURAL MODELING OF HYBRID REINFORCED CONCRETE</td>
<td>281</td>
</tr>
<tr>
<td>6.1. Introduction</td>
<td>281</td>
</tr>
<tr>
<td>6.2. Material Models</td>
<td>283</td>
</tr>
<tr>
<td>6.3. Derivation of Moment-Curvature Response</td>
<td>285</td>
</tr>
<tr>
<td>6.4. Parametric Studies of Material Parameters</td>
<td>289</td>
</tr>
<tr>
<td>6.5. Design Charts</td>
<td>292</td>
</tr>
<tr>
<td>6.5.1. Coefficient of Resistance</td>
<td>293</td>
</tr>
<tr>
<td>6.6. Simplified Design Equation for Moment Capacity</td>
<td>296</td>
</tr>
<tr>
<td>6.7. Minimum Flexural Reinforcement Ratio</td>
<td>297</td>
</tr>
<tr>
<td>6.8. Algorithm to Predict Load-Deflection Responses</td>
<td>303</td>
</tr>
<tr>
<td>6.9. Experimental Verifications of Flexural Model</td>
<td>303</td>
</tr>
<tr>
<td>6.10. Evaluation of Ultimate Strength</td>
<td>306</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6.11. Reduction Factors</td>
<td>307</td>
</tr>
<tr>
<td>6.12. Chapter Conclusions</td>
<td>309</td>
</tr>
<tr>
<td>7. DESIGN GUIDELINES FOR FIBER REINFORCED CONCRETE</td>
<td>354</td>
</tr>
<tr>
<td>7.1. Introduction</td>
<td>354</td>
</tr>
<tr>
<td>7.2. Strain-Softening Fiber Reinforced Concrete</td>
<td>355</td>
</tr>
<tr>
<td>7.2.1. Material Models and Parameters</td>
<td>355</td>
</tr>
<tr>
<td>7.2.1.1. Tension Model</td>
<td>356</td>
</tr>
<tr>
<td>7.2.1.2. Compression Model</td>
<td>358</td>
</tr>
<tr>
<td>7.2.1.3. Steel Model</td>
<td>359</td>
</tr>
<tr>
<td>7.2.2. Ultimate Moment Capacity</td>
<td>360</td>
</tr>
<tr>
<td>7.2.2.1. Fiber Reinforced Concrete System</td>
<td>360</td>
</tr>
<tr>
<td>7.2.2.2. Hybrid Reinforced Concrete System</td>
<td>362</td>
</tr>
<tr>
<td>7.2.3. Minimum Postpeak Tensile Strength for Shrinkage and Temperature</td>
<td>363</td>
</tr>
<tr>
<td>7.2.4. Shear Strength Capacity</td>
<td>365</td>
</tr>
<tr>
<td>7.2.5. Material Testing</td>
<td>367</td>
</tr>
<tr>
<td>7.2.6. Design Examples</td>
<td>368</td>
</tr>
<tr>
<td>7.2.6.1. Example 1: Slab on Grade</td>
<td>368</td>
</tr>
<tr>
<td>7.2.6.2. Example 2: Four-Span Floor Slab</td>
<td>371</td>
</tr>
<tr>
<td>7.2.6.3. Example 3: Wall Footing</td>
<td>373</td>
</tr>
<tr>
<td>7.2.7. Inverse Analysis of Load-Deflection Response</td>
<td>378</td>
</tr>
<tr>
<td>7.2.8. Discussions</td>
<td>381</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>7.3. Strain-Hardening Fiber Reinforced Concrete</td>
<td>382</td>
</tr>
<tr>
<td>7.3.1. Material Models and Parameters</td>
<td>383</td>
</tr>
<tr>
<td>7.3.2. Ultimate Moment Capacity</td>
<td>384</td>
</tr>
<tr>
<td>7.3.3. Model Comparisons</td>
<td>386</td>
</tr>
<tr>
<td>7.3.4. Design Examples</td>
<td>388</td>
</tr>
<tr>
<td>7.3.4.1. Example 1: Trough Cover</td>
<td>388</td>
</tr>
<tr>
<td>7.3.4.2. Example 2: Soil Container</td>
<td>391</td>
</tr>
<tr>
<td>7.3.5. Discussions</td>
<td>395</td>
</tr>
<tr>
<td>7.4. Chapter Conclusions</td>
<td>396</td>
</tr>
<tr>
<td>8. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS</td>
<td>416</td>
</tr>
<tr>
<td>8.1. Executive Summary and Conclusions</td>
<td>416</td>
</tr>
<tr>
<td>8.2. Summary of Fiber/Fabric Pullout Model and Recommendations</td>
<td>416</td>
</tr>
<tr>
<td>8.3. Summary of Tension Stiffening Model and Recommendations</td>
<td>419</td>
</tr>
<tr>
<td>8.4. Summary of Flexural Modeling and Recommendations</td>
<td>421</td>
</tr>
<tr>
<td>8.5. Summary of Hybrid Flexural Modeling and Recommendations</td>
<td>423</td>
</tr>
<tr>
<td>References</td>
<td>429</td>
</tr>
</tbody>
</table>
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1. Mix Designs of the Different Mixtures</td>
<td>109</td>
</tr>
<tr>
<td>3.2. Properties of the Fibers Made up the Fabrics</td>
<td>109</td>
</tr>
<tr>
<td>3.3. Material Parameters Used in Simulations</td>
<td>110</td>
</tr>
<tr>
<td>3.4. Key Parameters Obtained from Experiments and Simulations</td>
<td>111</td>
</tr>
<tr>
<td>3.5. Upper and Lower Bound Shear Strength Parameters for Series PP105</td>
<td>112</td>
</tr>
<tr>
<td>3.6. Numerical Models for Parametric Studies and the Simulation Results</td>
<td>112</td>
</tr>
<tr>
<td>3.7. Equivalent Bond Slip Parameters for a Smooth Yarn Pullout Model</td>
<td>113</td>
</tr>
<tr>
<td>3.8. Bond-Slip Parameters for a Finite Difference Fabric Pullout Model</td>
<td>113</td>
</tr>
<tr>
<td>4.1. Combination of Material Models Used in the Parametric Studies</td>
<td>180</td>
</tr>
<tr>
<td>5.1. Force Component and Moment Arm to Neutral Axis at Each Stage of Normalized Top Compressive Strain ($\lambda$)</td>
<td>247</td>
</tr>
<tr>
<td>5.2. Net force, Moment and Curvature for Each Stage of Normalized Top Compressive Strain ($\lambda$)</td>
<td>248</td>
</tr>
<tr>
<td>5.3. Neutral Axis Depth Ratio, Normalized Moment and Curvature for Each Stage of Normalized Top Compressive Strain ($\lambda$)</td>
<td>249</td>
</tr>
<tr>
<td>5.4. Details of the Mixes and Their Parameters Used in Simulation of Load-Deflection Response of Steel Fiber Reinforced Concrete</td>
<td>250</td>
</tr>
<tr>
<td>5.5. Normalized Height of Compression and Tension Zones for Each Stage of Normalized Tensile strain at Bottom Fiber ($\beta$)</td>
<td>251</td>
</tr>
</tbody>
</table>
Table                                                                                                           Page
5.6.  Normalized Stress at Vertices in the Stress Diagram for Each Stage of 
      Normalized Tensile Strain at Bottom Fiber ($\beta$) ............................................................... 252
5.7.  Normalized Force Component for Each Stage of Normalized Tensile Strain 
      at Bottom Fiber ($\beta$) ................................................................................................................. 253
5.8.  Normalized Moment Arm of Force Component for Each Stage of Normalized 
      Tensile Strain at Bottom Fiber ($\beta$) .......................................................................................... 254
5.9.  Equilibrium of Force, Moment and Curvature for Each Stage of Normalized 
      Tensile Strain at Bottom Fiber ($\beta$) .......................................................................................... 255
5.10. Neutral Axis Depth Ratio, Normalized Moment and Curvature for Each Stage 
      of Normalized Tensile Strain ($\beta$) ............................................................................................. 256
6.1.  Normalized Height of Compression and Tension Zones for Each Stage of 
      Normalized Compressive Strain at Top Fiber ($\lambda$) .............................................................. 317
6.2.  Normalized Stress at Vertices in the Stress Diagram for Each Stage of 
      Normalized Compressive Strain at Top Fiber ($\lambda$) .................................................................. 318
6.3.  Normalized Force Component for Each Stage of Normalized Compressive 
      Strain at Top Fiber ($\lambda$) ............................................................................................................ 319
6.4.  Normalized Moment Arm of Force Component for Each Stage of Normalized 
      Compressive Strain at Top Fiber ($\lambda$) ...................................................................................... 320
6.5.  Equilibrium of Force, Moment, Curvature and Stiffness for Each Stage of 
      Normalized Compressive Strain at Top Fiber ($\lambda$) ................................................................. 321
Table                                                                                                                               Page

6.6. Normalized Neutral Axis, Moment, Curvature and Stiffness for Each Stage of Normalized Compressive Strain ($\lambda$) ............................................................... 322

6.7. Parameters for Material models and Steel Rebar Locations for Using in Verification of the Simplified Equation for Ultimate Moment and Minimum Reinforcement Ratio .......................................................................................................................... 323

6.8. Beam Test Series ........................................................................................................................................................................ 324

6.9. Steel Fiber Reinforced Concrete Parameters for Updated RILEM Model ................................................................................................................................. 325

6.10. The Equivalent Steel Fiber Reinforced Concrete Parameters for the Proposed Model .................................................................................................................. 326

6.11. Comparison of Ultimate Load Capacity ................................................................................................................................................................. 327

7.1. Inverse Analysis Results of Load-Deflection Response of SFRC Mix .............. 403

7.2. Range of Material Parameters of GFRC Used in Simulations (7 Factors and 3 Levels)........................................................................................................ 404

xvii
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1. Application of fiber reinforced concrete: (a) Self-compacting fiber reinforced concrete; and (b) construction of strong floor in the Interdisciplinary Science and Technology Building Lab 2 (ISTB2) at Arizona State University.</td>
<td>16</td>
</tr>
<tr>
<td>1.2. Tensile stress strain behavior of specimens with AR- glass, E-Glass, and PP fabrics produced by pultrusion processing method as compared with conventional GFRC, and ECC materials.</td>
<td>17</td>
</tr>
<tr>
<td>1.3. Four types of fabrics used in the research program: (a) AR-glass; (b) Polypropylene (PP); (c) Polyvinyl Alcohol (PVA); and (d) Polyethylene (PE)...</td>
<td>18</td>
</tr>
<tr>
<td>1.4. Typical load-elongation responses of three replicate samples of AR-glass fabric</td>
<td>19</td>
</tr>
<tr>
<td>1.5. Hand lay-up method used in making of fabric reinforced cement composite from natural Sisal fibers</td>
<td>20</td>
</tr>
<tr>
<td>1.6. Pultrusion process used in making of a fabric reinforced cement composite from AR-glass fabrics</td>
<td>21</td>
</tr>
<tr>
<td>1.7. SEM micrographs of matrix penetration in between the openings of Polypropylene fabric: (a) hand lay-up method; (b) pultrusion method; and (c) the pullout responses of specimens prepared by these two methods</td>
<td>22</td>
</tr>
<tr>
<td>1.8. Textile reinforced concrete cladding at the Institute of Structural Concrete Aachen University (Hegger 2001; Hegger et al. 2001)</td>
<td>23</td>
</tr>
<tr>
<td>1.9. Permanent formwork for construction of reinforced concrete slab</td>
<td>24</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>1.10.</td>
<td>Micromechanics modeling of fabric–cement matrix interface: (a) SEM picture of the fabric-matrix interface; (b) bond model; (c) longitudinal yarn model; and (d) nonlinear spring model simulating anchorage resistant force provided by transverse yarns.</td>
</tr>
<tr>
<td>1.11.</td>
<td>Fiber pullout model using a beam on elastic foundation analogy</td>
</tr>
<tr>
<td>1.12.</td>
<td>Laminate fabric reinforced cement composite</td>
</tr>
<tr>
<td>1.13.</td>
<td>Meso-scale modeling of cracked tension specimen</td>
</tr>
<tr>
<td>1.14.</td>
<td>Generic material models and moment-curvature response for fiber and fabric reinforced cement composites: (a) strain-softening FRC model; (b) strain-softening/hardening FRC model</td>
</tr>
<tr>
<td>1.15.</td>
<td>Macro-scale modeling of structural applications: (a) bending test of fabric reinforced cement; (b) bending test of fiber reinforced concrete</td>
</tr>
<tr>
<td>2.1.</td>
<td>Fiber pullout model: (a) pull-push test; and (b) pull-pull test</td>
</tr>
<tr>
<td>2.2.</td>
<td>Bond stress versus slip relation (BSR): (a) Naaman et al. 1991, (b) Abrishami and Mitchell 1996 (c) Focacci 2000, (d) theoretical BSR; and (e) Banholzer et al. 2005</td>
</tr>
<tr>
<td>2.3.</td>
<td>Stress strain models for steel fiber reinforced concrete: (a) Lim et al. (1987), (b) Lok and Pei (1998), (c) Lok and Xiao (1999), (d) RILEM TC 162-TDF (2000); and (e) Elasaigh et al. (2004)</td>
</tr>
<tr>
<td>2.4.</td>
<td>Load-deflection curves deduced from flexural or direct tension tests</td>
</tr>
<tr>
<td>2.5.</td>
<td>Tensile stress strain models for SFRC according to Italian guideline</td>
</tr>
</tbody>
</table>
2.6. A schematic drawing of an elastic beam with a nonlinear cracked hinge model:
(a) three point beam test; (b) tensile stress strain and crack width model ......... 70

3.1. Scanning electron microscope (SEM) pictures of different fabric reinforced cement composites; (a) nonlinear geometry of longitudinal yarns and cross yarn junctions; (b) tortuous crack propagation; (c) matrix densification ........... 114

3.2. Effect of manufacturing techniques to the tensile stress strain response of fabric reinforced cement made of 2 layers of Saint Gobain Technical Fabrics.. 115

3.3. Pullout load slip response of Polypropylene (PP) embedded as a yarn an as a fabric ........................................................................................................................................ 116

3.4. Fabric pullout setup........................................................................................................... 117

3.5. Fabric pullout-slip response at difference stage of debonding ....................... 117

3.6. Schematic drawing of yarn pullout test: (a) side view of the upper half model;
(b) specimen cross section ................................................................................................. 118

3.7. Shear strength diagram ................................................................................................. 118

3.8. Shear stress and force distribution along the yarn: (a) Stage I (elastic response); (b) Stage II (nonlinear response); (c) Stage III (dynamic response);
(c.1) first dynamic responses (no sliding, $\Delta_d = 0$); (c.2) subsequent dynamic responses (when sliding occurs, $\Delta_d > 0$) ................................................................................................................. 119

3.9. Schematic description of the different fabrics: (a) bonded; (b) knitted;
(c) woven; (d) four fabric types used (number of yarns inside/outside cement block) ................................................................................................................................. 120

3.10. Casting of fabric pullout specimens........................................................................ 121
Figure 3.11. Fabric pullout specimens after cutting................................................................. 122

3.12. Experimental pullout responses prior and after adjustment for free fabric
length and initial deformation including the representative curve for each
control test series: (a) G105; (b) PP105; (c) PE105; (d) PVA103 ......................... 124

3.13. Representative pullout curves (averaged responses of 4 samples) of all 15 test
series used in simulations of the different fabrics: (a) AR-Glass (G) series;
(b) Polypropylene (PP) series; (c) Polyethylene (PE) series; (d) Polyvinyl
Alcohol (PVA) series ......................................................................................................... 126

3.14. Simulation of representative pullout curve for each control test series: (G105,
PP105, PE105 and PVA103) .......................................................................................... 127

3.15. Simulation of pullout responses for control series PP105 (average, lower and
upper bound). *The upper and lower bound responses are the averaged
response + 1 standard deviation.* .................................................................................. 128

3.16. Shear strength parameters: (a) averaged nominal shear strength from
experiments vs. maximum and frictional shear strength from best fit of
simulations; (b) averaged nominal shear strength from experiments with
standard deviation ........................................................................................................... 129

3.17. Correlation of shear strength parameters: nominal shear strength (τ_{nom}),
maximum shear strength (τ_{max}) and frictional shear strength (τ_{frc}). ...................... 130
3.18. Schematic drawing of fabric pullout mechanism; (a) interface bond-slip model; (b) longitudinal yarn stress-strain model; (c) spring force-slip model simulating anchorage strength at cross yarn junction; (d) fabric pullout specimen ................................. 131

3.19. Finite difference fabric pullout model; (a) discretized fabric pullout model under displacement control; (b)-(d) free body diagram of three typical nodes “A” - “C”............................................................................................................. 132

3.20. An algorithm for updating material stiffness and checking for convergence of local force deformation at each node for each imposed slip increment.......... 133


3.22. Material constitutive laws; (a) bond stress slip model; (b) longitudinal yarn stress strain model; (c) spring force slip model .................................................. 136

3.23. Compare distribution of slip, bond stress and yarn force along the embedded length; (a) and (b) imposed slip at the right end reaches 0.5 mm and 1.0 mm... 138

3.24. Parametric studies of material models to pullout load-slip responses; (a) the effect of bond strength; (b) the effect of slack; (c) the effect of spring strength 140

3.25. Comparison of pullout responses of steel rebar in normal strength concrete between analytical and numerical models; (a) the bond stress slip relation for both models; (b) the predicted pullout responses .................................................. 141

3.26. Simulation of fabric pullout using equivalent single yarn pullout model; (a) best fit bond slip parameters; (b) simulations of the experimental results .... 142
<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.27.</td>
<td>AR-glass fabric used in pullout test; (a) plain fabric; (b) fabric inside cement matrix</td>
</tr>
<tr>
<td>3.28.</td>
<td>Modeling of AR-Glass fabric reinforced cement; (a) pullout specimen; (b) representative numerical model</td>
</tr>
<tr>
<td>3.29.</td>
<td>Calibrated material parameters for AR-glass fabric reinforced cement; (a) best fit bond slip model; (b) best fit spring model</td>
</tr>
<tr>
<td>3.30.</td>
<td>Simulation of pullout response of AR-glass fabric; (a) cast sample, (b) pultrusion sample; (b) vacuum process sample; (d) mix with flyash sample</td>
</tr>
<tr>
<td>4.1.</td>
<td>Typical tension specimens made of four types of fabrics: (a) AR-glass; (b) Polypropylene (PP); (c) Polyvinyl Alcohol (PVA); (d) Polyethylene</td>
</tr>
<tr>
<td>4.2.</td>
<td>Typical tensile response of fabric reinforced cement based composites</td>
</tr>
<tr>
<td>4.3.</td>
<td>Mechanical behavior of a cracked fabric reinforced cement composite; (a) bond slip model; (b) longitudinal yarn stress strain model; (c) nonlinear spring model simulating anchorage strength at cross yarn junction; (d) matrix strength for cracking criterion</td>
</tr>
<tr>
<td>4.4.</td>
<td>Matrix strength distribution in tension specimen to achieve different crack patterns; (a) deterministic crack; (b) stochastic crack</td>
</tr>
<tr>
<td>4.5.</td>
<td>Finite difference model; (a) discretized fabric pullout model under load control (b) sign convention for slip and boundary conditions for force in yarn; (c) free body diagram of six representative nodes labeled as “A” – “F”</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>4.6.</td>
<td>Material models for parametric studies (a) bond slip model; (b) matrix strength, (c) yarn stress strain model; (d) nonlinear spring model</td>
</tr>
<tr>
<td>4.7.</td>
<td>Numerical base model using base material models (no options)</td>
</tr>
<tr>
<td>4.8.</td>
<td>Four normalized material responses of a numerical base model: (a) matrix stress distribution; (b) yarn stress distribution; (c) slip distribution; (d) shear stress distribution</td>
</tr>
<tr>
<td>4.9.</td>
<td>Normalized composite stress strain responses due to variation of; (a) bond strength; (b) matrix strength; (c) slack in longitudinal yarn; (d) spring strength</td>
</tr>
<tr>
<td>4.10.</td>
<td>Normalized averaged tensile stresses in cracked matrix as a function of normalized composite strain due to variation of; (a) bond strength; (b) matrix strength; (c) slack in longitudinal yarn; (d) spring strength</td>
</tr>
<tr>
<td>4.11.</td>
<td>Normalized averaged crack spacing as a function of normalized composite strain due to variation of; (a) bond strength; (b) matrix strength; (c) slack in longitudinal yarn; (d) spring strength</td>
</tr>
<tr>
<td>4.12.</td>
<td>Simulation of reinforced concrete tension members; (a) test setup; (b) bond slip model; (c) steel stress vs. elongation responses</td>
</tr>
<tr>
<td>4.13.</td>
<td>Tension specimen of AR-glass fabric reinforced concrete and its bond slip relationship; (a) specimen; (b) bond slip model</td>
</tr>
</tbody>
</table>
4.14. Simulation of AR-glass fabric reinforced concrete; (a) composite tensile stress strain (b) force components in composite; (c) tension stiffening in postcrack concrete; (d) average crack spacing and crack width as various stage of composite tensile strain. ......................................................................................... 204

4.15. Simulation of AR-glass fabric reinforced cement using bond and spring models; (a) bond and spring model; (b) best fit simulation of fabric pullout response; (c) effect of efficiency factor in prediction of composite tensile stress strain response; (d) crack spacing evolution as function of composite strain........................................................................................................................................ 206

5.1. Relationship between uniaxial tension test and flexural test of fabric reinforced cement: (a) comparison of experimental responses; and (b) comparison of cumulative probability distributions for LOP, MOR, BOP and UTS. ..................................................................................................................................... 257

5.2. Strain-softening fiber reinforced concrete model: (a) tension model; and (b) compression model.................................................................................................................................................. 258

5.3. Stress strain diagram at 3 stages of normalized top compressive strain $\lambda$: (a) elastic for compression and tension ($0 < \lambda < 1$); (b) elastic for compression but nonlinear for tension ($1 < \lambda < \omega$); and (c) plastic for compression and nonlinear for tension ($\lambda > \omega$)......................................................................................................... 259
<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4.</td>
<td>Normalized moment curvature diagrams and their approximate bilinear models: (a) deflection-hardening ((\mu &gt; \mu_{\text{crit}})); and (b) deflection-softening ((\mu &lt; \mu_{\text{crit}}))</td>
</tr>
<tr>
<td>5.5.</td>
<td>Comparison of normalized moment equations: (a) regression equation to determine the intersection point for normalized bilinear moment curvature diagram; (b) the normalized moment capacity between (M'_{\infty}) and (M'u)</td>
</tr>
<tr>
<td>5.6.</td>
<td>Three- and four-point bending test: (a) experimental setup; (b) moment distribution; (c) curvature distribution at bilinear cracking; (d) curvature distribution at ultimate moment for high normalized postpeak tensile strength ((\mu &gt; \mu_{\text{crit}})); (e) curvature distribution at ultimate moment for low normalized postpeak tensile strength ((\mu &lt; \mu_{\text{crit}}))</td>
</tr>
<tr>
<td>5.7.</td>
<td>Effect of normalized postpeak tensile strength (\mu) to: (a) neutral axis depth ratio (k); (b) normalized moment curvature diagram (M' - \phi')</td>
</tr>
<tr>
<td>5.8.</td>
<td>Effect of compressive yield strength ratio (\omega) to: (a) neutral axis depth ratio (k); (b) normalized moment curvature diagram (M' - \phi')</td>
</tr>
<tr>
<td>5.9.</td>
<td>Tension models and the predicted load-deflection response of steel fiber reinforced concrete under three-point bending: (a) tension model for H22; (b) load-deflection response for H22; (c) tension model for H3; (d) load-deflection response for H3</td>
</tr>
</tbody>
</table>
5.10. Tension models and the predicted load-deflection responses of steel fiber reinforced concrete under four-point bending: (a) tension model for H1; (b) load-deflection response for H1; (c) tension model for H21; (d) load-deflection response for H21

5.11. Material models for homogenized fiber reinforced concrete: (a) compression model; and (b) tension model

5.12. Stress strain diagram at different stages of normalized tensile strain at the bottom fiber ($\beta$):

5.13. (a) Moment curvature diagram and crack localization rules; and (b) four point bending test

5.14. Parametric study of a typical strain-softening material: the effect of parameters $\alpha$, $\mu$ and $\gamma$&$\omega$ to normalized moment curvature diagram

5.15. Parametric study of a typical strain-hardening materials: the effect of parameters $\alpha$, $\eta$ and $\gamma$&$\omega$ to normalized moment curvature diagram.

5.16. Simulation of a steel fiber reinforced concrete (SFRC): (a) and (b) tension model and flexural response of mix H1 ($V_f=0.5\%$); (c) and (d) tension model and flexural response of mix H21 ($V_f=1.0\%$)

5.17. Simulation of Engineered Cementitious Composite (ECC): (a) tension model; (b) flexural response

6.1. Material model for single reinforced concrete design: (a) compression model; (b) tension model; (c) steel model; (d) beam cross section
6.2. Strain and stress diagram at three stages of applied compressive strain at top fiber ($\lambda$): (a) stage 1: ($0 < \lambda < \lambda_{R1}$) elastic compression-elastic tension;
(b) stage 2: ($\lambda_{R1} < \lambda < \omega$) elastic compression – postpeak tension; (c) stage 3
($\omega < \lambda < \lambda_{cu}$) plastic compression – postpeak tension ........................................ 329

6.3. Material model for typical steel fiber reinforced concrete and rebar used in parametric studies: (a) concrete model; (b) steel model ............................................. 330

6.4. Parametric studies of neutral axis depth ratio for different levels of postpeak tensile strength parameter $\mu$ and reinforcement ratio $\rho_g$........................................... 333

6.5. Parametric studies of normalized moment curvature diagram for different levels of postpeak tensile strength parameter $\mu$ and reinforcement ratio $\rho_g$........ 334

6.6. Parametric studies of normalized stiffness degradation for different levels of postpeak tensile strength parameter $\mu$ and reinforcement ratio $\rho_g$...................... 336

6.7. Design chart of normalized ultimate moment capacity for different levels of post crack tensile strength $\mu$ and reinforcement ratio $\rho_g$ ................................. 337

6.8. Compare coefficient of resistance using ACI stress block method and the proposed equation: (a) for steel grade 275 MPa; (b) for steel grade 415 MPa... 339

6.9. Evaluation of the simplified equation for predicting normalized ultimate moment capacity: (a) between theoretical prediction and simplified equation;
(b) reliability of the simplified equation ............................................................. 340
6.10. Verification of minimum reinforcement ratio: (a) compare predicted normalized ultimate moment capacity at the minimum reinforcement; (b) the required minimum reinforcement for different level of postpeak tensile strength

6.11. Material stress strain model for Updated RILEM method: (a) tension model for SFRC; (b) compression model for SFRC; (c) steel model

6.12. Comparison of minimum reinforcement ratio; (a) conventional reinforced concrete beam; (b) FRC with steel rebar system

6.13. SFRC models for three levels of fiber contents (25 kg/m³, 50 kg/m³ and 75 kg/m³) used in SFRC beam series

6.14. SFRC models for three levels of fiber contents (25 kg/m³, 50 kg/m³ and 75 kg/m³) used in HRC beam series

6.15. Load-deflection responses of SFRC beams at three levels of fiber contents (25 kg/m³, 50 kg/m³ and 75 kg/m³)

7.1. Material model for single reinforced concrete design: (a) compression model; (b) tension model; (c) steel model; (d) beam cross section

7.2. Four-span floor slab

7.3. Wall footing
<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.4.</td>
<td>Inverse analysis of SFRC using Excel worksheet: (a) input data; (b) matching predicted load-deflection response to experimental post crack response of the average residual strength test method; (c)&amp;(d) tension and compression models obtained from inverse analysis</td>
</tr>
<tr>
<td>7.5.</td>
<td>Simplified material model for strain-hardening: (a) tension model; (b) compression model</td>
</tr>
<tr>
<td>7.6.</td>
<td>Range of material models generated: (a) tension model; (b) compression model</td>
</tr>
<tr>
<td>7.7.</td>
<td>Compare normalized ultimate moment capacity predicted by simplified model and full model: (a) elastic compression without yield limit; (b) elastic compression with limit at yielding</td>
</tr>
<tr>
<td>7.8.</td>
<td>Normalized design chart for GFRC</td>
</tr>
<tr>
<td>7.9.</td>
<td>Trough and cover in the industrial pavement: (a) geometry; (b) design chart</td>
</tr>
<tr>
<td>7.10.</td>
<td>Soil container: (a) dimension; (b) analysis model; (c) design chart</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

1.1. Overview
Concrete is a cementitious composite made of coarse granular materials embedded in hardened mortar. It can be molded to form any structural shapes to fulfill architect imagination and meet engineering requirement. When concrete is hardened, it becomes a durable material providing long lasting service life. The constituents necessary for making concrete such as rocks and sands are abundant in nature offering the resources to be used at reasonable prices. The concrete is strong in compression but weak in tension. The tensile strength is roughly 10% of the compressive strength. Therefore, the early forms of concrete structures were constructed in a compression mode such as masonry walls, arch bridges and cathedral domes.

To overcome the weakness mentioned above, concrete must be reinforced by the material which is strong in tension. Knowledge of reinforcing material has been known for thousands of years. The Egyptians made reinforced mud brick by mixing straw in a mud and drying it in the sun. This reinforced brick has been used as a main building material for centuries and it is still being used today. In modern world, a strong composite is made of concrete and steel which now becomes a standard term “reinforced concrete.” This practice started in the late eighteenth century. Negligible concrete tensile strength is assumed to be zero and steel is placed in the tension zone to carry all tension force applied to a designed member. Reinforced concrete structures are expected to have minor cracks in tension zone; yet, they still serve their duties for long service life with little requirements of maintenance.
Lessons learnt from series of devastations caused by earthquakes make engineers start realizing that reinforced concrete may not be sufficient. Due to the large deformation, concrete spalling removes lateral bracing material for the compression steel and leads to buckling of the reinforcement. The loss of concrete cover between tension steel and concrete matrix reduces interface area essential in force transfer mechanism of the composite action. Unpredictable loading and internal force redistribution generated during excitations require not only structural ductility that members are reinforced to withstand loads in specific direction, but also material ductility that material can sustain unexpected force redistribution. Tougher concrete matrix is preferable for structures in seismic zones that are at risk of violent loads. Another driving force for tougher concrete matrix comes from the need of more durable transportation infrastructures such as pavements and bridges that are subjected to repeated loads and exposed to extreme environments. As material fatigues and gets aged, the performance of structures deteriorates over time and the structures need constant maintenance, which may disrupt the services to publics. While scientists and engineers are gearing up developing high fatigue resistance materials, fiber and fabric retrofitting are currently utilized to strengthen the aging infrastructures, prolonging their service life. Carbon fiber wrapping around the bridge columns has been implemented in many seismic areas to increase confinement and enhance the load capacity for the next earthquakes to come.

Newly invented admixtures and knowledge of material microstructures have accelerated the concrete technology into the new level. High-range water-reducing admixtures (or superplasticizers) now become a common practice to improve rheological
property of fresh concrete which allows low water cement ratio to be used in producing high strength concrete. Large amount of fibers can be added to the mix while the handling of fresh concrete is still workable. Different kinds of pollzalanic materials derived from industrial wastes such as flyash and silicafume have been employed extensively to densify microstructures of concrete, leading to improvement in strength as well as durability. Large array of material compositions and admixtures can be selected and combined in order to create a specialized material or “tailor made concrete”. Extremely flowable concrete was mixed with steel fibers to create a variant of self compacting fiber reinforced concrete, which was used in a construction of strong floors of the Interdisciplinary Science and Technology Building Lab 2 (ISTB2) at Arizona State University (Fig. 1.1).

The concepts of high performance concrete and new types of cement based composites have caught research attention recently. These classes of engineered materials offer higher strength and ductility than normal concrete used in standard practices. Thus, their performances are well suit for the structures in seismic regions. In addition, the materials offer resilient fatigue resistance, attractive to be used in highways, bridges and industrial structures. Analytical models, numerical simulations and Scan Electron Microscope (SEM) have been employed to study the micromechanics of composite materials and the knowledge gained has been used to improve mechanical properties of the newly invented materials. One of the most successful advanced cement based material in the field is Engineered Cementitious Composite (ECC) invented at University of Michigan (Li et al 2001; Li 2003). ECC is a composite made of mortar and synthetic
fibers such as Polyvinyl Alcohol (PVA) or Polyethylene (PE) by detailed design of the microstructure in order to achieve maximum toughness and ductility. Unlike fiber reinforced concrete (FRC) that fracture localization occurs immediately after the first crack is formed, propagation of initial crack in ECC is resisted by fiber bridging mechanism. Substantial energy is required to drive further crack growth or else the secondary cracks must form. Single crack localization is therefore shifted to multiple distributed cracking mechanisms, leading to macroscopic pseudo strain-hardening behavior. With large strain capacity, ECC is well suited for the applications exposed to cyclic loading such as bridge decks and retrofit of walls subject to seismic loading.

On the verge of energy crisis and the impact of global warming we are facing today, the concept of sustainability has been incorporated as a strategic development in many countries around the world. Civil engineers who are mainly responsible for using natural resources such as rocks, woods and steel for making houses, buildings and infrastructures, play a major role in the sustainability cycle. Construction activities are dealing with handling of heavy materials, consuming huge amount of energy and polluting the environments. At Arizona State University, the material research program sponsored by US National Science Foundation has focused on the development of an alternative fabric reinforced cement composites, which is a composite of synthetic fabrics and concrete. The goal is to create a new class of sustainable construction materials that consume less natural resource but offer superior tensile strength as well as ductility for being used as load bearing structural members.
1.2. Fabric Reinforced Cement Composites

Fabric reinforced cement composites constitute a recent addition to the family of cement based composite materials (Brameshuber et al. 2003; Kruger et al. 2003). Various research groups have developed a wealth of recent information pertaining to the methodologies, properties, and areas of applications for fabric reinforced cement composites (Meyer and Vilkner 2003; Brameshuber 2002; Jesse and Curbach 2004). Very encouraging tensile strength and ductility responses have been observed by various experimental programs with tensile strength of the order of 20 MPa and tensile strain capacity of 3-5% (Peled and Mobasher 2005; Reinhardt et al. 2003). Successful manufacturing of composite laminates has clearly demonstrated the benefits of the pultrusion process for several fabric types. Fig. 1.2 shows the superior tensile behavior of pultruded composites, greater than four folds in strength and an order of magnitude in toughness as compared with the conventional Glass Fiber Reinforced Concrete Systems (GFRC) and Engineered Cementitious Composite (ECC) (Li et al. 2001; Li 2003).

1.2.1. Fibers and Fabrics

From fracture mechanics concept, strength of materials can be substantially increased by eliminating flaw in bulk materials. Making material in a form of fiber that the microstructure is aligned with less disruption and flaw size is minimized results in a significant gain in material strength. Multiple fibers are normally bundled to make a yarn to prevent fiber breakage and provide convenient form of usage. Fibers or yarns can be either fabricated to form a textile structures for being used as a thin sheet or they are chopped to made discrete fibers for being used as the reinforcement in fiber reinforced
concrete. Fig. 1.3 shows four types of fabrics used in the research: AR-glass, Polypropylene (PP), Polyvinyl Alcohol (PVA) and Polyethylene (PE). The textures of these fabrics are as follows: bonded AR-glass, warp knitted weft insertion PP, and woven PE and PVA. In bonded fabrics a perpendicular set of yarns are attached together at the junction points using a polymeric adhesive. In warp knitted weft insertion fabrics the yarns in the warp direction are knitted into stitches to assemble with straight yarns used in the weft direction. In woven fabrics the warp and the weft (fill) yarns pass over and under each other and the reinforcing yarns are in a crimped shape.

Apparent modulus of fabrics are largely depending on the types of materials and fabrication methods used to form the fabrics; ranging from the highest modulus of AR-glass around 58 GPa, PP 4.9 GPa, PE 0.96 GPa to as low as 0.63 GPA of PVA fabrics. Modulus of fabrics is general lower than the modulus of individual filament made up of the fabrics due to crimping effect of multifilament in a yarn bundle that gradually slip and stretch as the load is applied. Despite the impressive tensile strength, the failure mode of the fibers and fabrics are general brittle as shown by the rapid drop in postpeak strength of the AR-glass fiber in Fig. 1.4.

1.2.2. Concrete Matrix

Cement based matrix for the fabrics are normally either mortar or fine grain aggregate concrete. There is little difference in term of mechanics of brittle materials among various types of concrete matrix. Therefore the simplest form of cement paste was used in the current research to study the fundamental mechanical properties of fabric reinforced cement composites. Several cement paste mixtures were studied in order to identify the
most compatible matrix to be used with fabrics. The main compositions of the cement pastes used are Portland cement type I, flyash, silica fume and superplasticizer.

**1.2.3. Manufacturing Methods**

Fabric reinforced cement composite used in the research were prepared by two main manufacturing methods: hand lay-up (cast) and pultrusion. As the name imply, the hand lay-up method is simply done by placing the fabric in between cement paste matrix as demonstrated by the preparation of a specimen using natural “Sisal” fiber in Fig. 1.5. Longitudinal fibers are stitched together to form a fabric sheet to be used with a mortar containing high volume of metakaolin, which results in a brick color. A clean layer of mortar is poured at the base and the fabric is then placed on the top. Additional mortar is poured at a top layer and the surface is finished by hand screed. This primitive method yield low productivity and inconsistent quality; yet, the simple technique can be used in many rural areas to create an affordable material for housing. In order to increase the productivity and obtain more uniform products, a pultrusion technique is introduced. Fig. 1.6 describes the pultrusion process of AR-glass fabric-cement composites. The fabric is first pre-impregnated in a cement bath to increase the penetration of matrix into the openings of the fabrics. Then the fabric is pulled though the roller to remove excess cement paste and the impregnated fabrics are roved over a spandrel to create a thin composite laminates. In fabric reinforced cement composite, the matrix penetration into multifilament yarns is a critical factor that can lead to the improvement in strength and ductility. Fig. 1.7(a) shows the example of poor cement penetration into a PP fabric manufactured by a hand lay-up method while Fig. 1.7(b) shows a better penetration with
pultrusion technique. Higher pullout load capacity from the test result of PP fabrics shown in Fig. 1.7(c) indicates that the bond property of specimen manufactured by pultrusion technique performs better than the primitive hand lay-up method.

1.2.4. Commercial Products

Fabric reinforced cement composite or another name textile reinforced concrete (TRC) has been primarily developed in Germany. The composite is manufactured in a form of thin sheet with high specific strength and ductility, attractive to be used in light weight thin shell structures. Due to fine aggregate used in mixture composition, the exposed surface of TRC appears high quality sharp edge suitable for decorative work at exterior buildings. At this stage, the knowledge of strength capacity in different modes of loading is still limited; however, the applications such as cladding panels and permanent formwork system have already been implemented in practices.

Fig. 1.8 shows cladding panels made of TRC for the extension of the building at the Institute of Structural Concrete Aachen University in Germany (Hegger 2001; Hegger et al. 2001). The dimensions of the panels were 2685 mm x 325 mm x 25 mm, much thinner than ordinary steel reinforced concrete panel, which is in the range of 70 mm - 100 mm. The weight reduction of using this type of panels results in a simpler anchorage system. Another potential growth area of this TRC is a permanent form work as shown in Fig. 1.9 which is intended to be used in a construction of conventional steel reinforced concrete floors. The TRC formwork is capable of carrying its own weight during erections and construction. After the concrete hardens, the formwork acts as an extra reinforcement in the composite floor system.
1.3. Objectives

The objective of this research was to explain mechanical the behaviors of fiber and the fabric reinforced cement composites which were subjected to various design variables such as fabric types, manufacturing technique, admixtures and material constituents, observed in different modes of material testing, which includes pullout, tension and bending. Several experimental programs were accomplished by previous graduate students (Sueki 2003; Pahilajani 2004; Singla 2004). Some of the test data were analyzed by the previous students and some were reanalyzed by the theoretical models developed in this dissertation.

On the theoretical parts, the program focuses on muti-scale modeling of fabric reinforced cement composites, starting from microstructural level to explain the pullout mechanism of fabric and cement matrix; meso-scale modeling of yarn bridging force in tension specimen to simulate crack evolution and obtain tensile stress strain characteristics and macro-scale modeling of flexural response of structural members. Unlike the high-end composites used in aerospace industries, the composite materials used in civil engineering applications must be simple in order to minimize fabrication cost, analysis and design. Therefore, structures can be built at reasonable prices. With this regard, the composites are normally made in a homogenized form and the complexity of material properties can be viewed as equivalently uniaxial material models. The theoretical models are primarily developed for fabric reinforced cement composites and then extended to fiber reinforced concrete, conventional reinforced concrete and the hybrid of the two systems. In addition, the design guidelines for using various composite
systems have been proposed.

1.3.1. Micromechanics Modeling of Fabric-Cement Matrix Interface

It has been widely accepted that the use of continuous fabrics in cement matrix has better load transfer mechanism than the discrete fibers. The excellent bonding comes from the undisrupted shear stress transfer at the interface of longitudinal yarn and mechanical anchorage provided by transverse yarns (fill yarns). Since the pullout responses of various fabric structures (woven, warp knitted and bonded) show resemblance to the pullout response of a straight yarn (Zastrau 2003; Peled and Bentur 2003; Peled et al. 2006), straight fiber pullout models have often been used to characterize the equivalent bond properties of fabric reinforced cement. Sueki et al. (2007) modified the Naaman (1991a,b) pullout model and used it to characterize the equivalent bond properties of the fabric reinforced cement that varies in fabric types, mixture compositions, processing methods and embedded lengths.

For better simulation of bonding between fabrics and cement paste, the micromechanics pullout model as shown in Fig. 1.10 consisting of three constitutive laws for interface, longitudinal yarn and nonlinear spring simulating the anchorage provided by the transverse yarn is developed. The concept of using N-linear piecewise functions to describe bond-slip relation by Banholzer (2005a) was adopted in the current research. All the constitutive laws used in the fabric pullout model are described by multilinear segments providing a great flexibility and powerful tool in simulations of different mechanical aspects of the composite constituents. The effect of slack commonly found in the composites made by hand lay-up method (Singla 2004) can be modeled by adding an
experimental observed slack at the beginning of the longitudinal yarn stress stain model. The effect of the anchorage provided by transverse yarns was studied by Sueki (2003), in which the transverse yarn was assumed to behave as a beam on elastic foundation (Fig. 1.11). The resistant force contributed by the beam is added to the pullout force without compatibility of the displacement at the junction. The analogy of beam on elastic foundation is replaced with nonlinear spring concept to enforce slip continuity across the junction. Finally, the finite difference method is used to integrate the three key constituents to solve the nonlinear pullout problem.

1.3.2. Mesomachanics Modeling of Crack Evolution in Tension Specimen

A meso-scale modeling is used to link the responses at the micromechanics level to structural responses in the macroscopic level. Several desirable material characteristics that enhance structural performance can be studied at this meso-scale level. One of the most interesting properties of the fabric reinforced cement composites is the apparent tensile stress strain response obtained from tension test. In order to explain crack evolution in fabric reinforced cement composite, Mobasher et al. (2006a) proposed a model based on the ply discount method of classical laminate theory (Talreja 1986), where the fabrics and matrix were modeled as discrete phases as shown in Fig. 1.12. The algorithm employed an incremental approach to update the ply stiffness. The change of crack spacing and stiffness degradation were accounted by means of damaged parameters at different tensile strain levels. The effect of transverse yarn in fabric structure that increases pullout resistance was modeled by a beam on elastic foundation analogy. The model can capture the trend of increasing pullout strength and staggering drop in the
postpeak region; however, the saw tooth response in the prepeak response was exaggerated and not observed in the experiment.

For better understanding of the tensile behavior of composites, a one-dimensional model consists of series of pullout crack segments is used to simulate crack evolution and obtain the apparent tensile stress strain response. Fig. 1.13 shows a meso-scale modeling of tension specimen made of fabric and cement paste. The fabric pullout model developed in the micromechanics level is used to simulate the yarn bridging force at crack locations in the tension specimen. As the number of cracks increases and crack spacing decreases, an individual crack segment is solved as a pullout problem and the analysis results of all cracked segments are then combined to represent the tensile behavior of the specimen. The apparent load and elongation obtained from the simulation are normalized with the cross sectional area and gage length to represent nominal tensile stress strain characteristic of the materials.

1.3.3. Macromachanics Modeling of Structural Applications

A variety of cementitious composite systems has been developed over the years, all aiming to increase tensile strength, enhance material toughness and reduce crack width. Though the systems are called differently, they share some similarities in material responses: the compressive strength is several times stronger than the tensile strength and the shape of the compressive stress strain response is similar to that of plain concrete with improved ductility in the postpeak. The only main difference is the tensile stress strain response, which depends mainly on the reinforcing systems and volume fraction used.

For practical modeling of structural applications in a macro-scale, the complicate
mechanical behaviors of two distinct phases in composites are simplified to homogenized material responses, which hold true for the composites that the fibers are uniformly distributed in the matrix (Soranakom and Mobasher 2006a). The homogenized material responses can be obtained directly from the experiments or indirectly from simulation at the meso-scale level. With a concept of homogenization, a unified approach based on moment-curvature relationship is possible which will provide a rational and practical way to predict and design flexural members of various composite systems.

Two generic material models as depicted in Fig. 1.14 (a and b) can be used to approximate the compressive and tensile responses of different types of fiber and fabric reinforced cement composites, including strain-softening and strain-hardening materials. Each generic model is used to derive the closed form solution for moment-curvature relationship as shown in Fig. 1.14(c). The relationship is linearly elastic up to the cracking limit and the postcrack response can be either deflection-softening or deflection-hardening, depending on the level of postcrack tensile strength in the tension model. Load-deflection responses of structural members such as fiber reinforced cement beam [Fig. 1.15(a)] and fabric reinforced concrete beam [Fig. 1.15(b)] can be predicted by using the same concept of moment-curvature relationship and moment area method. In addition, the moment-curvature relationship can be used as an input parameter in nonlinear finite element modeling to simulate a more complex structure subjected to various geometrical and loading conditions.

1.3.4. Fiber Reinforced Concrete with Conventional Steel Rebar

Conventional steel reinforced concrete (continuous system) and fiber reinforced concrete
(discrete system) have been widely used in civil engineering applications. The continuous reinforcement is more efficient in terms of materials used, but the lay down of the reinforcement is labor-intensive and time-consuming. On the other hand, fiber reinforced concrete in a form of ready mix is more convenient in construction process but less efficient in terms of material strength. The hybrid reinforced concrete using of these two systems is a compromised solution between the efficiency and ease of construction. In several areas where pre-fabricated steel wire mesh is available, the use of the wire mesh can significantly reduces the amount of discrete fibers required for fiber reinforced concrete providing a cost effective construction process.

The concept of using moment-curvature relationship and moment area method to predict flexural responses of structural members is more practical than performing 3D nonlinear finite element analysis which is very computational intensive. Therefore, the concept of moment-curvature relationship is extended to the case of hybrid system by adding a steel model to the fiber reinforced concrete section in the formulation of moment-curvature relationship.

1.4. Conclusions

The research program presented in this study makes substantial contributions to the field of fiber and fabric reinforced cement based composites, which are highly valuable to many researchers as well as engineers. The research is summarized with the proposed design guideline for fiber reinforced concrete and hybrid reinforced concrete systems. A brief guideline for strain-hardening GFRC is also proposed. The rational guidelines
provide systematic approaches of material characterizations to obtain design parameters and the use of design parameters to determine the member capacities.
Fig. 1.1. Application of fiber reinforced concrete: (a) Self-compacting fiber reinforced concrete; and (b) construction of strong floor in the Interdisciplinary Science and Technology Building Lab 2 (ISTB2) at Arizona State University.
Fig. 1.2. Tensile stress strain behavior of specimens with AR- glass, E-Glass, and PP fabrics produced by pultrusion processing method as compared with conventional GFRC, and ECC materials.
Fig. 1.3. Four types of fabrics used in the research program: (a) AR-glass; (b) Polypropylene (PP); (c) Polyvinyl Alcohol (PVA); and (d) Polyethylene (PE)
Fig. 1.4. Typical load-elongation responses of three replicate samples of AR-glass fabric
Fig. 1.5. Hand lay-up method used in making of fabric reinforced cement composite from natural Sisal fibers
**Fig. 1.6.** Protrusion process used in making of a fabric reinforced cement composite from AR-glass fabrics
Fig. 1.7. SEM micrographs of matrix penetration in between the openings of Polypropylene fabric: (a) hand lay-up method; (b) pultrusion method; and (c) the pullout responses of specimens prepared by these two methods
Fig. 1.8. Textile reinforced concrete cladding at the Institute of Structural Concrete Aachen University (Hegger 2001; Hegger et al. 2001)
Fig. 1.9. Permanent formwork for construction of reinforced concrete slab
Fig. 1.10. Micromechanics modeling of fabric–cement matrix interface: (a) SEM picture of the fabric–matrix interface; (b) bond model; (c) longitudinal yarn model; and (d) nonlinear spring model simulating anchorage resistant force provided by transverse yarns.
Fig. 1.11. Fiber pullout model using a beam on elastic foundation analogy

Fig. 1.12. Laminate fabric reinforced cement composite
Fig. 1.13. Meso-scale modeling of cracked tension specimen
Fig. 1.14. Generic material models and moment-curvature response for fiber and fabric reinforced cement composites: (a) strain-softening FRC model; (b) strain-softening/hardening FRC model
Fig. 1.15. Macro-scale modeling of structural applications: (a) bending test of fabric reinforced cement; (b) bending test of fiber reinforced concrete
CHAPTER 2

LITERATURE REVIEW

2.1. Interface Properties between Fiber and Matrix

In civil engineering, two commonly used test methods to investigate the bond properties between fiber and cement matrix are pull-push test [Fig. 2.1(a)] and pull-pull test [Fig. 2.1(b)]. Since there are no analytical models for fabric (grid structure) pull-out test available in literatures, the smooth fiber pullout models are reviewed instead. The basic ideas from the existing models can be used to develop a fabric pullout model for fiber/fabric reinforced cement composites. Analytical models for fiber pullout tests can be categorized into three approaches: perfect interface model, fracture mechanical model, and cohesive interface model.

2.1.1. Perfect Interface Model (Stress Approach)

Cox (1952) developed the so-called perfect interface model in which the bonding between matrix and fiber was perfect; thus, the displacements and tractions were continuous at the interface. The interface can be viewed as an axis-symmetry problem which simplifies the 3D problem to 2D problem while the elastic equations for an axis-symmetric stress state can be used to formulate the pullout model as done by Timoshenko and Goodier (1984); McCartney (1989); and Nairn (1997). However, their explicit solutions were extremely difficult to obtain and in many cases they were extremely complex. A further simplification from 2D problem to 1D problem was done by assuming $|du_r/dx| \ll |du_x/dr|$ so that $\gamma_{xr} \simeq du_x/dr$, where $u$ is the deformation, subscripts $r$ and $x$ refer to the radial and longitudinal direction, respectively, and $\gamma_{xr}$ is the shear strain between $r-x$ coordinate. The stress in axial direction is independent of radial
direction and \( u_r \) is a function of \( r \) only. Using these assumptions, Nayfeh (1977) derived the second order differential equation for the fiber force distribution in the fiber for the pull-push test [Fig. 2.1(a)]. The interface between matrix and fiber was defined by a so-called shear lag parameter which was dependent on the Young’s modulus and shear modulus of the fiber and matrix as well as the volume fractions of the two materials. In composites, when shear stress exceeds the shear strength of the interface, the debonded zone is created and the perfect bond assumption is violated. Nairn (1997) applied the concept of constant post-elastic bond strength to simulate the frictional stress in the debonded zone. Due to the idealization of the composite (sharp edge) and the assumed linear elastic material properties with a perfect bond in the intact region, the derivation resulted in stress singularity at the crack tip which was not expected to exist in the real structures. Stang (1985) and Shah and Ouyang (1991) criticized the use of perfect interface bond model to predict the debonding behavior and proposed the fracture mechanics approach instead.

### 2.1.2. Fracture Mechanical Model (Energy Approach)

Unlike the stress approach that the debonding occurs when shear stress exceeds the shear strength limit, a fracture at the interface of fiber and matrix initiates when the energy stores in the system exceeds the energy limit. The release of energy creates the crack surfaces along the fiber direction. Griffith (1920) was the first to recognize the energy required for crack propagation that can be equated to the increase of surface energy. The fracture mechanic approach employs the law of energy conservation which can be expressed as
\[ W = U + K_E + U_S \]  \hspace{1cm} (2.1)

where \( W \) is external energy, \( U \) is the internal energy which consists of elastic and inelastic deformation, \( K_E \) is the kinetic energy, and \( U_S \) is the surface energy due to crack propagation. In static or quasi-static pullout test, \( K_E \) is negligible and can be ignored; thus, the energy equilibrium can be rewritten as done by Lue (1992).

\[ W = U_e + U_f + U_S \]  \hspace{1cm} (2.2)

where \( U_e \) is the elastic strain energy in the bonded region and \( U_f \) is the inelastic energy due to friction in the debonded region. The interface is separated into a bonded region where two materials are intact and debonded region in which damages occur. The shear stress in debonded region is assumed to constant frictional bond strength. Several researchers (Budiansky et al. 1986; Gao et al. 1988; Kim et al. 1992a,b; Liu and Narin 1999; Leung and Yang 2002) have used this assumption to derive expressions for the energy release rate \( G \). Besides the constant frictional bond model, other realistic models proposed to describe bond behaviors at interface are possible. However, the analytical forms are very difficult to achieve or even impossible.

**2.1.3. Cohesive Interface Model (Stress Approach)**

The concept of two material bonds perfectly at the interface may not be realistic for some composite materials. It has been observed in steel fiber reinforced concrete that there was a thin interphase existing between concrete matrix and a reinforcing fiber (Pinchin and Tabor 2000; Bentur et al. 1986; Igarashi et al. 1996). Bentur et al. (1986) identified this transition zone consists of calcium hydroxide, porous layer of calcium silicate hydrates and ettringite, extending from the surface of the fiber to about 50 \( \mu \text{m} \) and the zone had
different material properties from the bulk material. The strength of this zone is generally lower than the common cement by about 30%. The very thin thickness of the interphase was renamed to an interface (with zero thickness). This approximation results in the displacement discontinuity between the matrix and reinforcing element. The shear stress at interface is now only a function of local slip and shall be called bond stress versus slip relation (BSR).

With this concept, the cement based matrix is linked to the fiber by an independent BSR model and the pullout boundary value problem can be expressed by a second order differential equations. Naaman et al. (1991a, b) proposed an elastic with constant postcrack bond strength model [Fig. 2.2(a)] and derived the second order differential equations in terms of fiber force and constant material parameters. Slightly different from the Naaman’s model, Abrishami and Mitchell (1996) proposed another BSR model that had an additional linear softening right after the crack as shown in Fig. 2.2(b). Later Focacci et al. (2000) proposed a nonlinear bond stress versus slip relationship superposing a modified exponential expression and a linear decreasing function as shown in Fig. 2.2(c). Alternatively, the second order differential equation for pullout problems can be formulated in terms of local bond stress $\tau$, local slip $s$ and material parameters (Stang and Shah 1990; Russo et al. 1990; Focacci et al. 2000). The BSR models in Fig. 2.2(a)-(c) share the common features that they are described by a certain number of material parameters. The more realistic BSR model [Fig. 2.2(d)] that begins with elastic response and continues with nonlinear portion up to the peak, followed by the softening postpeak response is very difficult or impossible to derive for the analytical equation.
Banholzer (2005a) introduced an N-linear piecewise BSR model as shown in Fig. 2.2(e) to approximate the realistic bond behavior described in Fig. 2.2(d). The derivation used two equilibrium equations, a compatibility equation, and Hook’s law together with two initial conditions, to formulate an initial boundary value problem. For any given N-linear piecewise BSR model, the pullout load–slip response can be generated. In the reverse direction, any pullout load–slip response can be recovered to the N-linear piecewise BSR model by an inverse boundary value analysis Banholzer (2005b). However, some oscillations can occur in the recovering process and the average curve passed through the center of the oscillations was used as the best estimated N-linear piecewise BSR model.

2.1.4. Approach for Fabric Reinforced Pullout Model

According to the three approaches discussed in the previous sections, it can be seen that the fracture mechanics and cohesive interface model can simulate the bond failure mechanism at interphase, independent of matrix and fiber. This provides a great flexibility in modeling of the actual interphase existing between the matrix and fiber. In fracture mechanics approach, the energy release rate is used to describe how much the system can store the energy before releasing it to create new crack surfaces along the fiber. The concept of energy balance is less familiar to general engineers. On the other hand, the cohesive interface model provides a physical meaning that describes the process of pulling a fiber out of the matrix, generating shear stress along a very thin layer of interphase (or zero thickness of interface). When stress exceeds the critical value, the debonded zone is developed and the strength decreases as a function of slip. The cohesive interface model can also be linked to the well established fracture mechanics. The amount
of energy release rate $G$ can be obtained by integrating the area under curve of bond-slip relationship. Therefore, the cohesive interface model shall be used as a based approach to develop a fabric pullout model for fiber/fabric reinforced cement composites.

2.1.5. Fabric Pullout Model

It has been widely accepted that the use of continuous fabric (grid structure) has a better load transfer mechanism than randomly distributed discrete fibers in the matrix. The excellent bonding comes from the undisrupted load transfer along the longitudinal fiber direction and the mechanical anchorage provided by transverse yarns. However, little work has been done in this area to develop a model that can explain the effect of transverse yarns. Since the pullout load versus slip response of various fabric structures (woven, warp knitted and bonded) all show resemblance to the pullout response from straight yarn (Zastrau et al. 2003; Peled and Bentur 2003; Peled et al. 2006), pullout models derived for a straight yarn have often been used in material characterizations to obtain equivalents bond properties of the fabric reinforced cement.

Sujivorakul et al. (2000) developed a pullout model aiming to study the effect of an anchorage of a hook-ended fiber. The first fiber was attached to the second fiber by a joining spring. The first segment was intended to simulate a physical straight fiber and the second segment for a fictitious anchorage. The joining spring acts as a force transmitter between the two segments. If the spring is very weak, only the first fiber carries the load. This is similar to a pullout of a single fiber. If the spring is indefinitely stiff, the pulling force is increased due to the additional length of the second fiber which acts as anchorage. Even though the model cannot physically describe the hook-ended
fibers, it provides a fictitious resistant due to the anchorage at the end of the straight portion of the first fiber. This idea can be applied to the fabric pullout mechanism that the longitudinal yarn is attached to springs at equally spaced cross yarn junctions, simulating the resistant provided by transverse yarns.

The most realistic analytical pullout model for fabric reinforced cement composites to date was studied by Sueki (2003). The effect of transverse yarn was modeled by a beam on elastic foundation analogy and the effect was then added to the longitudinal yarn pullout resistant load. The effect of transverse yarn was limited to the ultimate strength of the fabric junction obtained experimentally. The model can simulate the pullout mechanism of a fabric that contains yarns in both orthogonal directions. However, the assumption of beam on elastic foundation oversimplifies the possibility of material nonlinearity of the cement paste subjected to high concentrated load. Furthermore, the contribution of transverse yarn to the pullout load in longitudinal direction was combined without displacement compatibility at the junctions.

From the literature survey discussed above, the cohesive interface approach was selected for the development of a fabric pullout model. N-linear piecewise BSR concept proposed by Banholzer (2005a,b) that provides a great flexibility in modeling of bond behaviors at the interface is extended to all the constitutive laws required for the new invented fabric pullout model.

2.2. Tension Responses of Fiber Reinforced Cement Composites

Cracking in reinforced cement based composites is unavoidable due to the low tensile strength of concrete matrix. The width of crack, however, needs to be controlled within
an acceptable level to protect reinforcement from corrosion and to preserve aesthetic appearance of structures. Postcrack stiffness of reinforced concrete members is much less than the initial elastic stiffness and may cause excessive deformation at service loads. Therefore, the ability of reinforced concrete composites in carrying load after cracking is a very important issue. Despite the concrete lost most of its tensile strength at crack locations, it is still be able to carry some tension force between two parallel cracks that causes the material response to appear stiffer than the expected response of an assumed zero concrete tensile strength. This enhanced stiffness is called “tension stiffening” which is depending on cracking mechanisms in reinforced members such as crack spacing, crack width, and the bond between reinforcing materials and matrix. This phenomenon is observed for all reinforcing materials embedded in concrete matrix such as steel rebar, fiber reinforced plastic (FRP), yarn, and fabric. Tension stiffening is normally evaluated by three main approaches: experimental, analytical, and numerical.

2.2.1 Empirical Approach

It is an essential step in material research to conduct experimental programs and establish empirical equations for particular sets of the factors investigated. The experimental results provide valuable information of material behaviors and the empirical equations can explain the relationship between the input variables and material responses measured. The results must be interpreted according to the specific range of test parameters.

Rizkalla and Hwang (1984) conducted an extensive experimental program on reinforced concrete tension member containing longitudinal and transverse steel reinforcement. Major design variables included ratio of reinforcement, concrete cover,
concrete thickness, and spacing between transverse reinforcement. Normal strength concrete (NSC) of 34.5 MPa and two grades of deformed steel bar of 300 MPa and 400 MPa were used in the specimen preparations. Based on the test results, they proposed an algorithm to predict average crack spacing, number of cracks and crack width. The predicted number of cracks showed good agreement with the measured values while the predicted crack width showed quite scattering results compared to the measured crack width. However, most of the predicted values were within the boundaries defined by the minimum and maximum crack width.

As concrete changes from normal strength concrete (NSC) to high strength concrete (HSC), the compressive stress strain response becomes more brittle and less ductility. However, the dense micro structure of HSC improves bond strength between concrete steel. These opposite factors raise the question how HSC affects the reinforced concrete structures. Various research groups showed contradictions of their test results. The use of HSC reduced crack width by 20% - 40% at the stabilized cracking phase and it delayed the formation of cracks at higher load (Jaccoud et al. 1993; Farra 1995; Hamouine 1996). In contrast, the crack width due to bending of HSC beams at service load showed 50% greater than the identical beams made of NSC (Manalip 1994; Roure 1985, 1990). According to Hoang (1985), no significant changes could be detected between the cracking behavior of HSC and NSC beams. Lorrain et al. (1998) investigated the cracking behavior of both NSC and HSC tension ties under short term loading. The test parameters were mechanical strength of concrete, reinforcement ratio and the yield strength of deformed steel bar. The test results showed that using HSC greatly improved
tension stiffening in the crack formation phase and tended to decrease during the stabilized crack due to the bond damaging at steel–HSC interface. Crack spacing depended mainly on the reinforcement ratio and it was less affected by the increase of concrete strength. The use of HSC (100 MPa) in tension specimen allowed a reduction of crack width from 10% to 60%.

In evaluation of tension stiffening effect, shrinkage that pre-shortens axial strain must be taken into account (Bischoff 2001); otherwise, high performance concrete that has higher shrinkage or the member that contains more steel reinforcement ratio will cause perceived reduction in the first cracking strength and tension stiffening effect. Corrected results indicated that tension stiffening was independent of the reinforcing steel ratio (1% - 2% used in the test) and continued to decrease after cracking has stabilized. Bischoff (2003) studied the postcrack behavior of reinforced concrete tension member made of plain concrete and SFRC. The author used load sharing approach to determine the concrete contribution to the load capacity. The concrete contribution was then normalized with the concrete cracking load to obtain the bond factor which can be used as material property of cracked concrete. Unlike the plain concrete that only uncracked portion between two parallel cracks carry tensile force by means of bond mechanism, SFRC can carry tensile stress between cracks as well as across the crack face. The experimental final crack spacing of SFRC was about half of those plain concrete.

2.2.2 Analytical Approach

Analytical approach provides a rational way to explain physical behaviors of crack evolution in tension specimens. The relationship between bond stress and crack patterns
is normally used as the basis to formulate the analytical models. Several analytical models have been developed so far.

Somayaji and Shah (1981) proposed the use of an assumed exponential bond stress function in the second differential equation for slip distribution. By integrating the equation twice and consequentially applying boundary conditions, the slip distribution at the concrete-steel interface can be obtained.

Floegl and Mang (1982) proposed an approximate relationship between the average bond stress, the average tension strain in the reinforcing bars, the ultimate compressive strength of concrete, and the hydrostatic compressive stress in concrete. The relationship was then used in a nonlinear finite element analysis program.

Gupta and Maestrini (1990) used a bilinear bond stress-slip relation to formulate the equations for predicting tension-stiffening behaviors of a cracked reinforced concrete specimen. In simplification, bond parameters were eliminated completely to obtain a normalized tension stiffening model.

Instead of using an assumed bond stress distribution or deriving the tension stiffening model explicitly based on the bond slip relation, Kwak and Song (2002) assumed a concrete strain distribution with an \( n \)th order polynomial function in which the order \( n \) is identified by iteratively solving on the basis of the energy equilibrium before and after cracking. Once the order \( n \) is identified, the calculations of concrete strain, steel strain, slip, force in concrete and steel are straightforward.

Khalfallah (2006) presented the tension stiffening model based on repartition functions of loads, strains and slip between steel and surrounding concrete. Cracking
strength of reinforced concrete member was expressed as a function of the maximum and average stress of concrete along the length and constant parameters, depending on the size of the specimen in which it makes cracking strength to increase as the number of crack increases.

In fabric reinforced cement composite systems, Mobasher et al. (2006a) proposed the composite laminate theory with an incremental stress approach to simulate tensile stress strain behaviors in pre and post crack responses. A damage parameter was used to simulate the softening of cement paste stiffness in the Bend Over Point (BOP) region (Shah and Ouyang 1991). Experimental results of crack evolution and stiffness degradation were related to the applied tensile strain and the experimental pullout stiffness was used to simulate the crack bridging force across the matrix cracks.

Based on the well known ACK model proposed by Aveston et al. (1971) and Aveston and Kelly (1973), Cuypers and Wastiels (2006) incorporated two - parameter Weibull distribution [Curtin et al. (1998, 1999)] to simulate stochastic cracking strength of the concrete matrix. The use of Weibull parameters obtained from three point bending test of plain concrete did not perform satisfactory results in prediction of tensile stress strain behaviors. Thus, the parameters determined by the curve fitting of the simulation curve to experimental response were recommended.

2.2.3 Numerical Approach

When material behaviors are complicate, several parameters are required in the development of the mathematical models. Derivations using several parameters would result in lengthy equations or even not possible. The remedy is to divide the complex
problem into several small tasks and solve them numerically. Finite element method is the most widely used numerical tool in the field of engineering. It has been employed to simulate bond mechanism, cracking and tensile behavior of brittle materials.

Holler et al. (2004) presented a finite element model for textile-reinforced concrete under tension and bending loads. The reinforced concrete composite was modeled as a homogenized material consisting of plan stress layers of concrete and uniaxial layers of yarn reinforcement. The finite element based on the Reissner-Mindlin shell theory for finite rotation was used (Menzel 1996). Concrete was described by a nonlinear elastoplastic damage model with material softening (Darwin and Pecknold 1977). The tensile behavior of concrete prior to cracking was assumed to be linear elastic. Once the concrete cracked, its stiffness was set to zero and the residual concrete tensile strength was assigned to the yarn by means of tension stiffening obtained by the iterative solving the force equilibrium within a textile-reinforced concrete tension bar over an average crack spacing. Either a single bond interface at matrix-yarn or two bond interfaces at matrix-outer filaments and outer filaments-internal filaments can be used in the calibration process to obtain the yarn tensile stress strain model including tension stiffening effect. The simulations of the proposed model showed good agreements with the tension and bending test results.

Instead of using shell theory, Haussler-Combe and Hartig (2007) modeled textile-reinforced concrete as a uniaxial reinforced concrete, which consisted of concrete matrix, sleeve filaments and core filaments. Bond at concrete-sleeve interface can be different from the bond at sleeve-core interface to simulate force transfer mechanisms in the
composite. The perimeter of the yarn was calculated from the assumed circular cross section and modified by a roughness coefficient and shape coefficient to realistically describe the actual contact perimeter. Finite element method was employed in numerical investigations of textile reinforced concrete behaviors starting from the ideal case; followed by microscopic deficiency mechanisms such as broken bond in longitudinal direction, waviness slip in the yarn bundle and premature failure of filaments.

Hegger et al. (2006) simplified the bond layer model proposed by Konrad et al. (2004) into two-subroving model. The bond layer model varies the highest bond quality at the outer filaments to the poorest quality at the core filaments. The model also includes the delayed activation that accounts for the waviness of the filaments. The mesomechanics simulations of the bond layer model was conducted to obtain a yarn pullout-slip response and the response was then inversely analyzed to obtain bond parameters for the two-subroving model, which were the bond slip relation at the interface of matrix-sleeve filaments and the bond slip relation at the interface of sleeve-core filaments and the initial delayed strain for the core filaments stress strain model.

Among the three approaches that can be used to study the tensile behaviors of fiber/fabric reinforced cement composites, the numerical method is the most powerful approach that can be used to characterize bond properties at fiber-matrix interface and simulate crack evolution in the composites. However, nonlinear finite element modeling of composite structures is very computational expensive. For a simple task such as uniaxial reinforced concrete problem, a computation less intensive finite difference method can be as effective as the finite element. Thus, the finite difference method was
selected in the development of the tension stiffening model for fiber/fabric reinforced cement composites.

2.3. Flexural Modeling of Fiber Reinforced Cement Composites

Fiber reinforced cement composite consists of concrete matrix and discrete/continuous fibers. The presence of fibers in the mix provides crack bridging forces, resulting in enhancement of tensile strength and ductility. Various techniques have been used to create different types of fiber reinforced composites. All aims to improve tensile capacity, increase fracture toughness, and minimize crack width of the concrete matrix. Steel fiber reinforced concrete (SFRC) was made by adding steel fibers into the concrete mix (Umekawa and Nakazawa 1970). The presence of randomly distributed fibers in the matrix strengthens the flaws and prevents it from initiating crack in material. If cracks occur, the fibers act as crack arresters to hold the cracks from opening. With addition of fibers, the postcrack response is significantly improved; yet, it marginally increases the cracking tensile strength. Using the same approach, but at much longer fibers and higher volume fractions than standard SFRC, slurry infiltrated concrete, SIMCON (Krstulovic-Opara and Malak 1997; Bayasi and Zeng 1997) and Ductal (Chanvillard and Rigaud 2003) have exhibited significant enhancement in tensile strength and postcrack response. In contrast with the discrete reinforcement, the use of continuous fibers such as ferrocement (FRC) (Naaman and Shah 1971) provides a more efficient way to increase tensile capacity. The superior performance comes from the fact that smaller diameter of wire meshes has more contact area per unit volume to transfer more force to matrix. The transverse wire also acts as a mechanical anchorage adding more resistance to the pullout
force. Besides using steel as reinforcement, other materials such as polypropylene (PP) (Naaman and Shah 1984) and alkali resistant (AR) glass (Marikunte et al. 1997) have also shown improvement in tensile capacity and ductility. More recently, various textile reinforced cements (TRC) (Swamy and Hussin 1989; Peled et al. 1998; Peled and Mobasher 2005) have also exhibited distributed cracking mechanisms and strain-hardening behaviors (Peled et al. 1999).

Despite of the names called for different composite systems, they share similarity in material constitutive behaviors: the compressive strength is several times stronger than the tensile strength such that the flexural behavior is strongly controlled by the tensile capacity. The shape of compressive stress strain model is fairly similar to that of plain concrete. The prepeak response is slightly affected by the fiber content while the ductility of the postpeak is either improved due to distributed cracking. One the other hand, the tensile response of each system can be significantly different from one another due to reinforcing mechanism and fiber contents. Generally speaking, the material characteristics of various fiber reinforced cement composites (SFRC, SIMCON, DUCTAL, FRC and TRC) are relatively close to the steel fiber reinforced concrete with different fiber contents; thus, SFRC can serve as a based model for all composites.

A strain compatibility analysis of layered beam can be used to generate moment curvature response for predicting flexural behavior of all materials mention above (Park and Paulay 1975). However, the analysis requires discretization of beam cross section, iterative solving for neutral axis and numerical integration to determine moment and curvature at each imposed strain at extreme fiber. Closed form solutions that can be used
to generate moment curvature diagram, predict load-deflection response and determine flexural capacity are preferable. Since SFRC can be used as a based model for various composite systems, the review on its relevant research is valuable. There have been a number of closed form solutions for various proposed SFRC models as shown in Fig. 2.3.

Fig. 2.3(a) shows the material models for low fiber content SFRC proposed by Lim et al. (1987a,b). An elastic-perfectly plastic model was used to describe a compressive response. The rule of mixture, fiber orientation, and length efficiency factors were employed to determine the elastic compressive curve. The compressive yield stress can be assumed in the range of 0.45 to 1.0 of the cylinder compressive strength (Swift and Smith 1978; Swamy 1985; Nishioka et al. 1975) and the crushing compressive strain was taken as 0.004 (Swamy 1985). A linear elastic model with constant residual strength was used to describe the tensile behavior. The precrack tensile response is obtained by the rule of mixture, fiber orientation, and length efficiency factors while the postcrack response was obtained by pullout strength, fiber orientation, and length efficiency factors. Soroushian and Bayasi (1991) criticized the use of law of mixture in prediction of the precrack behavior at relatively low steel fiber contents in which the effect of steel fibers at this stage is negligible. Lim et al. (1987) used his empirical stress strain models to derive closed form solutions for generating moment curvature diagram. However, a ratio of tension to compression zone is needed in the expression, which can be found by solving a quadratic equation and selecting the correct one from the two solutions. The ultimate moment was derived for two cases: elastic compression - bilinear tension and plastic compression - bilinear tension. With further simplification of moment curvature
diagram to bilinear models, the closed form solutions for load-deflection responses and toughness index of three and four point bending test can be derived.

Lok and Pei (1998) proposed a more realistic stress-strain model for steel fiber reinforced concrete as depicted in Fig. 2.3(b). The compression response consists of a parabolic curve from the origin to the peak and continues with a constant plastic stress up to the ultimate compressive strain of 0.0035. It is assumed that the initial tangent modulus of the compression and tension were the same. The tension response was modeled by 3 segments: a parabolic curve from the origin to the cracking tensile strength, linear softening response from the cracking strength to a transition point \( (\varepsilon^*_2, \sigma^*_2) \) where the dynamic fiber pullout was fully established, and gradually linear decreasing from a transition point to zero at ultimate tensile strain. Due to uncertainty of materials and measurements, the ultimate tensile strain is taken as \( 2 \times 10^4 \) \( \mu \)str (Craig et al. 1987), \( 1.45 \times 10^4 \) \( \mu \)str (Chung and Mai 1987) or \( 10 \times 10^4 \) \( \mu \)str (Sakai and Nakamura 1986). Similar to Lim et al. (1987), the expressions for generating moment curvature diagram at different stages of applied top compressive strain were provided, but they are required to solve a higher order equation and select a correct solution for the tensile strain \( \varepsilon^*_2 \) to be used in the given expressions. Several cases for solution of \( \varepsilon^*_2 \) could arise and were discussed in Pei (1996). The chart to determine an approximate ultimate moment capacity based on a simplified secondary tensile model was also developed for the rapid assessment of flexural strength. The prediction was relatively poor but on a conservative side (average experiment/prediction = 1.33) due to several assumptions made in the derivations.
Lok and Xiao (1999) simplified the material model used in Lok and Pei (1998) as shown in Fig. 2.3(c). The third linear segment in tensile stress-strain model was changed to a constant residual tensile strength. Closed form solutions for moment and curvature at cracking were derived explicitly. When compressive strain at extreme top fiber is still in elastic range, the moment and curvature at ultimate can be derived explicitly. Instead of solving higher order equation, the entire moment curvature diagram can be constructed by successively imposing a small compressive strain from zero to the failure and solving for the incremental tensile strain at extreme fiber. The extreme tensile strain was then updated and used to calculate curvature and moment. With approximation that the maximum moment occurs at transit strain $\varepsilon_{t1}$, a slightly conservative moment capacity was derived explicitly. The ultimate flexural stress can be established to the tensile cracking strength obtained from direct tension test and dynamic bond strength from pullout test.

Vandewalle (2000, 2002, 2003) proposed a stress-strain relationship shown in Fig. 2.3(d) which was obtained from the empirical relationship to the load-deflection response of three point bending test of a notched specimen. It requires strain compatibility analysis of layered beam cross section to determine the moment capacity.

Elsaigh et al. (2004) proposed a generic material model for SFRC as shown in Fig. 2.3(e), which used a simple elastic-perfectly plastic model for compression and trilinear model for tension. Numerical algorithm to generate moment-curvature diagram and predict load-deflection with Matcad 7 were also proposed. The stress strain models
for particular mixtures can be obtained by inverse analysis of load-deflection response of beam test.

Alternatively, discrete fiber reinforced concrete can be modeled using stress crack width relationship as done by Zhang and Stang (1998). For a bending test with a major crack at the center, the relationship between stress crack width and load-deflection/crack mouth opening (CMOD) responses can be established. This approach is less generalized than the stress-strain approach because the location of the crack and its propagation path must be known in advance. It is suitable for tracking one major crack propagate in a structural member such as a notched beam specimen under in three point bending test. For multiple crack propagation, only numerical method such as finite element can be used in the analysis. The change of boundary along the crack fronts requires auto adaptive meshing which is too complicate for general uses in material characterization.

Stress crack-width approach can be linked to the pullout mechanism at crack face. Oh et al. (2005) and Prudencio et al. (2006) applied probabilistic approach to simulate the pullout forces of steel fibers from the crack face. Load-deflection/CMOD responses were derived by assuming a major crack initiates and propagates at a center of a beam and the beam rotated rigidly around the support. Pullout slip responses of fibers were superimposed to stress-crack width relationship of plain concrete to simulate resistant of the crack in fiber reinforced concrete.

From the literature reviews discussed above, they imply that there is a need to develop a generic material model to predict flexural response of discrete/continuous fiber reinforced cement composites. Closed form solutions are preferred over an iterative
algorithm for the ease of implementation in spreadsheet as commonly used in design offices. Due to the ductility gained by adding fiber to brittle concrete, the normalized quantities with respect to their values at first cracking are also favorable, which can be used as another measure for expressing material toughness. The stress strain approach, which is more familiar to practice engineers and more versatile in analysis of cracked structure than the stress crack width approach, will be used to develop the closed form solution.

2.4. Design Guides for Fiber Reinforced Concrete

Steel fiber reinforced concrete has been used in constructions of concrete slabs and beams since the early seventies. Today, the use of SFRC is still limited in a few market sectors, mainly due to the lack of standard guidelines for material characterizations and design procedures. In order to promote the use of different types of fiber reinforced concrete that has better ductility than normal concrete, several committees have been set up to provide technical guidelines of using the materials. The summary of design methodology for flexural members of each code of practice will be briefly discussed.

In the United States, the available design guide for flexural members made of SFRC is based on the work of Swamy et al. (1974). The regression analysis of test data was used to determine the coefficient for the cracking strength $\sigma_{cr}$ and ultimate flexural strength $\sigma_{c,max}$,

$$\sigma_{cr} = 0.843\sigma_{m,cr}V_m + 425V_f \frac{l_f}{d_f}$$  

(2.3)
\[ \sigma_{c,\text{max}} = 0.97\sigma_{m,cr}V_m + 494V_f \frac{lf}{df} \]  

(2.4)

where \(\sigma_{m,cr}\) is flexural strength of the unreinforced matrix, \(V_m\) is the matrix volume fraction, \(V_f\) is the fiber volume fraction and \(lf\) is the length of fiber and \(df\) is the diameter of fiber. Particular type of fibers and nature of concrete was not specified in the guideline. These two equations have been found to overpredict the flexural strength by up to 50%. When SFRC is used with steel rebar, Henager and Doherty (1976) proposed the tensile stress block similar to the concept of compressive stress block in the ultimate strength design described in the ACI building Code (ACI-318, 2005).

\[ M_n = A_s f_{sy} \left( d - \frac{a}{2} \right) + \sigma_t b \left( h - e \right) \left( \frac{h}{2} + \frac{e}{2} + \frac{a}{2} \right) \]  

(2.5)

and

\[ \sigma_t = 1.12V_f \frac{lf}{df} F_{be} \]  

(2.6)

where \(M_n\) is the nominal moment capacity, \(A_s\) is the area of steel reinforcement, \(f_{sy}\) is the yield strength of steel, \(d\) is the depth of steel reinforcement, \(a\) is the depth of the compressive stress block, \(\sigma_t\) is the tensile strength in tensile stress block, \(b\) is the width of beam, \(h\) is the total depth of beam, \(e\) is the depth of tensile stress block, \(F_{be}\) is the bond efficiency factor (1.0 – 1.2). There have been a number of investigations on the shear capacity of beams with FRC, however, the design equation for shear capacity is still debating. The modern guidelines for FRC beyond those described above are being prepared by the ACI Committee 544.
In European Union countries, the RILEM committee TC162-TDF was set up in April 1995 to develop the testing and design procedures for SFRC (Vandewalle et al. 2000, 2002a, 2003). The committee proposed three-point bending test of a notched beam specimen for material characterization. The tensile stress strain relationship for SFRC can be correlated to the flexural strength at limit proportionality \( f_{ct,L} \) and two residual flexural strengths: \( f_{R,1} \) at CMOD = 0.5 mm and \( f_{R,4} \) at CMOD = 3.5 mm. The tensile stress strain diagram \( \sigma - \varepsilon \), Young’s modulus \( E \) and size dependent safety factor \( \kappa_h \) can be obtained as follows:

\[
\sigma_1 = 0.7 f_{fctm, fl} \frac{(1600 - d)}{1000}; \quad \varepsilon_1 = \frac{\sigma_1}{E_c} \tag{2.7}
\]

\[
\sigma_2 = 0.45 f_{R,1} \kappa_h; \quad \varepsilon_2 = \varepsilon_1 + 0.001 \tag{2.8}
\]

\[
\sigma_3 = 0.37 f_{R,4} \kappa_h; \quad \varepsilon_3 = 0.025 \tag{2.9}
\]

and

\[
E = 9500(f_{cm})^{1/3} \tag{2.10}
\]

\[
\kappa_h = 1 - 0.6 \frac{(h - 125)}{475} \tag{2.11}
\]

where \( f_{fctm, fl} = f_{ct,L} \) if bending test is performed; otherwise, \( f_{fctm, fl} \) can be calculated from the compressive strength according to ENV 1992-1-1, \( f_{cm} \) is the uniaxial cylinder compressive strength, \( h \) is the full height of the tested beam. The SI unit for stresses, strains and dimensions are MPa, mm/mm and mm, respectively. The compression response is described by a parabolic-rectangular stress strain model in which the parabolic curve reaches the peak stress at \( 0.85 f_{fcd} \) at the compressive strain of 0.002 and
remains plastic until reaches the ultimate compressive strain at 0.0035. The strain compatibility of a layered beam cross section is required to determine the ultimate moment capacity.

The German guideline for design flexural members is very much similar to that of RILEM (Teutsch 2004). They both use the strain compatibility analysis of a layered beam cross section to determine bending capacity. The main difference is that the shape of the tensile stress-strain model used in the German guideline has a small plastic region after cracking before the stress linearly drops to the final residual tensile strength at the ultimate tensile strain of 0.025, similar to the RILEM model.

The practice of FRC in United Kingdom traditionally follows the Japanese Standard; however, it has been recently shifted toward the test and design methodology proposed by the RILEM (Barr and Lee 2004). The Japanese standard JCI-SF4 (1984) outlines the determination of an equivalent flexural strength for a tested beam as follows:

$$\sigma_b = \frac{T_{JCI} L}{\delta_{\text{limit}} bd^2}$$

(2.12)

where $T_{JCI}$ is the energy absorbed by the beam up to a load point deflection of $L/150$, $L$ is the test span and $\delta_{\text{limit}}$ is the deflection limit of $L/150$. The toughness is measured by the equivalent strength ratio $R_{e,3}$ as the residual cracking load at $L/150$ mm to the cracking load. It should also be noted that $L/150 = 3$ mm for a standard beam size of 150 mm x 150 mm x 450 mm. The toughness of FRC is included in the design procedure of concrete floor slab (Concrete Society 20003). The residual positive moment is calculated as
\[ M_{\text{pos}} = \frac{f_{ctk,fl} R_{e,3} h^2}{6\gamma_c} \] (2.13)

where \( f_{ctk,fl} \) is the characteristic flexural strength of plain concrete, \( h \) is the slab thickness and \( \gamma_c \) is the partial safety factor for concrete. The limiting design criterion is assumed at the onset of cracking at the top of the slab. Since fibers negligible increases cracking strength of concrete, the negative bending moment capacity is calculated as

\[ M_{\text{neg}} = \frac{f_{ctk,fl} h^2}{6\gamma_c} \] (2.14)

The Italian guideline for design of FRC members starts with material classification: Type “A” (softening behavior), Type “B” (hardening behavior) and Type “C” (over-resistant behavior), based on load-deflection curves deduced from flexural or direct tension test as shown in Fig. 2.4 (di Prisco et al. 2004). The idealized tensile stress strain diagram is shown in Fig. 2.5 and the parameters defined as

\[ \epsilon_1 = \frac{f_{1\text{td}}}{E} \] (2.15)

\[ \varepsilon_{Fu} = \begin{cases} \frac{\delta_d}{l_{cs}} & \text{for Type "A"} \\ \frac{\delta_u}{l_{cs}} & \text{for Type "B" and "C"} \end{cases} \] (2.16)

where \( f_{1\text{td}} \) is the designed initial tensile strength, \( E \) is the Young’s modulus, \( \delta_d \) is the limit (ultimate) design deflection required to the SFRC, \( \delta_u \) is the deflection corresponding to \( P_{\text{max}} \) and \( l_{cs} \) is the characteristic length. Similar to RILEM and German guidelines, strain
compatibility analysis is required to identify the moment capacity. Alternatively, a simplified rectangular stress block of compression and tension zone can be used.

From the literature reviews discussed above, it can be noticed that the design guidelines for fiber reinforced concrete proposed by various committees are in favor of stress-strain approach over stress-crack width approach. Several empirical equations are used as parts of design procedures in order to correlate the responses measured from the test to the design parameters. These equations may limit the applicability of the design guides to a particular fiber system that was used in the establishment of equations. The most striking problem in material characterizations are the definitions of material parameters invented by researchers. In addition to material parameters, an enormous number of subscripts and superscripts are used to distinguish different aspects of the test and their theoretical assumptions. The development of modern design guides should be simple, consistent, and general, covering different types of discrete and continuous fiber systems. This can be accomplished by using generic material models to describe the material behaviors of fiber/fabric reinforced cement composites and derive for their closed form solutions for generating moment-curvature diagram. The moment-curvature diagram can be implemented in the algorithm to back calculate material parameters from the load-deflection response of bending tests. Finally, the closed form solutions are simplified to obtain design equations to estimate flexural capacity of a structural member.

2.5. The Relationship between Stress Strain and Stress Crack Width Model

Tensile properties of concrete materials are normally presented in an average sense by means of tensile stress-strain relationship despite of the possibilities of discontinuities at
crack locations. From scientific point of view, tensile properties of concrete should be characterized by fracture mechanics approach resulting in material parameters for fracture toughness and crack propagation criterion. The stress-crack width relationship is used to determine fracture toughness and to simulate crack propagation in concrete materials. The area under stress-crack width relationship describes the fracture energy $G_f$ required to create crack surface in concrete materials. Therefore, the review of fracture mechanics in concrete, stress-crack width, and its relationship to tensile stress strain model is worth mentioned.

Fracture mechanics was pioneered by Alan Arnold Griffith (1920) a British aeronautical engineer. He hypothesized that brittle materials contain elliptical microcracks which introduce high stress concentrations near their tips and he proposed the relationship between crack length $a$, surface energy of traction-free crack surfaces $(2\gamma)$, Young’s modulus $E$ and applied fracture stress $\sigma_f$ as follows:

$$\sigma^2_f = \frac{2\gamma s E}{\pi a}$$

(2.17)

His theory is a foundation of linear elastic fracture mechanics LEFM having applications in brittle materials such as fracture in glass. Later on, Irwin (1957) extended the Griffith’s work by introducing line crack that has two singularities at the crack tips and proposed the crack growth criterion stating that: the strain energy release rate $G$ must be greater than the critical work $G_c$ in order to advance crack growth. Furthermore, he showed that by using Westergaard’s method, the stress field in the area of crack tip is completely determined by the stress singularity $K_i$
\[ K_I = \sigma \sqrt{\pi a} \]  

(2.18)

where the subscript \( I \) refers to the first mode of crack opening. The application of LEFM was applied to rock fracture problems for underground constructions such as mines, excavations, and tunnels. McClintock and Walsh (1962) introduced the friction between crack face, whereas Kaplan (1961) focused on the possibility of using LEFM to concrete materials. Another milestone of fracture mechanics was crack growth in metal. Rice (1968) proposed the so-called \( J \)-integral which is the path independent integral around the crack tip that can be used as a crack growth criterion for nonlinear materials. It was proven that \( J \)-integral was equivalent to the energy release rate \( G \) in LEFM. Nevertheless the \( J \)-approach did not give correct results for concrete-like materials because the unloading curve characteristic of brittle material and the poorly defined location of the crack tip.

The turning point of fracture mechanics for concrete materials was starting by the development of cohesive crack models. Barenblatt (1959) and Dugdale (1960) introduced the cohesive force in the crack tip region within the limit of elastic theory. This force is applied to a small region so that the crack faces close smoothly. Hillerborg (1976) proposed the cohesive crack model which was widely accepted and exploited for analyzing cracks in concrete. His model describes the crack consisting of two zones: the fictitious crack in which crack faces are controlled by a tension softening law and the real crack that is stress free at the crack surfaces. However, its opening is controlled by the closing pressure in the fictitious crack zone. The model does not have the stress concentration at the tip of fictitious crack. Since then, a wide verity of cohesive crack
models have been proposed for modeling crack propagation in plain concrete and fiber reinforced concrete. An exponential decay function or simplified bilinear model is suitable for plain concrete while multilinear segments are suitable for fiber reinforced concrete.

Zhang and Li (2004) employed fracture mechanics parameters to explain cracking behavior of three-point bending test. Stress intensity factor due to remote loading \( K_a \) is superposed with stress intensity factor due to cracking bridging force \( K_b \) to calculate the crack tip stress intensity factor \( K_I \). The model relies on fracture toughness of hardened cement paste \( K_{IC} \) and the stress crack-width relationship of the material as the fundamental material parameters for model input. However, the model prediction is affected by the initial flaw size \( a_0 \) which can only be determined from the experimental result of flexural stress and cracking length for a given \( K_{IC} \). The assumed \( a_0 \) of 0.5 mm and \( K_{IC} \) of 0.2 MPa-m\(^{1/2} \) were shown as a very good fit for the simulation of flexural stress-crack mouth opening displacement (CMOD) of steel fiber reinforced concrete.

Fracture mechanics approach can be simplified to an elastic beam with nonlinear cracked hinge model as shown in Fig. 2.6. The concept is very close to moment curvature diagram in which the plastic length \( L_p \) is used to convert strain to elongation and curvature to rotation. With this approach, fracture mechanics parameters such as fracture toughness \( K_{IC} \), and related geometrical functions used in the calculation of stress intensity factor \( K_I \) are eliminated. Ulfkjaer et al. (1995) assumed postcrack capacity of concrete with a linear softening model and derived closed form solutions for normalized moment-rotation response. They found that the plastic length of a half beam depth \( (L_p = 0.5h) \)
correlated well with the results obtained from numerical model. Stang and Olesen (1998, 2000) used bilinear stress-crack width relationship to derive the analytical solutions for FRC. Olesen (2001) extend the model to include the case of beam subjected to normal force and presented the closed form solutions for normalized moment-rotation and load-deflection response of a three point bending test. Iyengar et al. (2002) employed a generalized power law function to describe postcrack tensile strength of concrete and derived an analytical model for predicting normalized moment-rotation response. He also studied the effect of the size of plastic length $L_p = 0.1h – h$ and proposed the steps to determine this length. Since the power law function is more suitable for plain concrete rather than FRC, the determination of plastic length for FRC is not clear.

In material characterization, the stress-crack width relationship can be directly obtained from the direct tension test; however, the test procedure requires fully fixity at the grips which is rather cumbersome to conduct. Alternatively, three-point notched beam test which is easier to perform are commonly used to evaluate tensile properties and the stress-crack width relationship can be determined by means of inverse analysis using the load-deflection or load-CMOD response. de-Oliveira-e-Sousa and Ravindra (2006) employed the analytical formulation derived by Stang and Olesen (1998,2000) in the inverse analysis algorithm using optimizer to minimize the residual error between the predicted and experimental responses. Weighting function was implemented in the algorithm to give unequal importance of the fitting interval in different portions of the load-CMOD response. The stress-crack width relationship obtained directly from direct
tension test showed a good agreement with the relationship obtained from the inverse analysis.

Barros et al. (2005) conducted a large number of three point notched beam tests according to RILEM TC 162-TDF recommendations and reevaluated the relationship between the residual flexural strength $f_R$ and the RILEM tensile stress strain model. They employed the elastic beam with nonlinear cracked hinge model with inverse analysis algorithm to back calculate stress-strain and stress-crack width relationship from the bending test results. Based on the best fit tensile parameters, the finding suggested that the coefficients used in the RILEM method (Eqs. 2.7- 2.9) need to be changed significantly. He further investigated the relationship between postcrack tensile strain $\varepsilon_{pcr}$ and crack width $w$ and found that they are related by the plastic length as $\varepsilon_{pcr} = w/L_p$. Noted that Barros ignored the small elastic elongation $\sigma_{pcr}/E$ in the postcrack region when calculating the postcrack tensile strain. In a practical range, the crack width is less than 3 mm and the best fit value of $L_p$ is $h_{sp}/2$ (or 62.5 mm). For overall crack width up to 10 mm, the best fit value of $L_p$ is about 74 mm which is close to the one obtained in a previous research by Barros and Sena-Cruz (2001). However, the correlation between postcrack tensile strain and crack width showed large scatter results due to the influence of many parameters; the most important is the structure size. Future research is required to investigate this issue.

From the literature reviews in this section, it is sufficient to conclude that both tensile stress-strain and tensile stress-crack width relationship can be obtained by inverse analysis of the beam test results using the elastic beam with nonlinear cracked hinge
model. Stress-strain and stress-crack width models are related by the assumed plastic length $L_p$

$$w = \left( \varepsilon_{pcr} - \frac{\sigma_{pcr}}{E} \right) L_p$$

(2.19)

where the negligible elastic elongation $\sigma_{pcr}/E$ can be ignored. As mentioned by Barros et al. (2005), the ratio of $w/\varepsilon_{pcr}$ was rather scattered due to many factors, it is possible that $L_p$ may not be unique but depends on several parameters that are not yet known. However, the value of $L_p = 0.5h$ is widely accepted in the characterization of concrete and FRC materials (Ulfkjaer et al. 1995, Pedesen 1996, Iyengar et al. 1998, Balaji 2000, Vandewalle et al. 2002b).

**Notation**

*The following symbols are used in this paper:*

- $A_s$ = area of steel reinforcement
- $a$ = depth of the compressive stress block or crack length
- $a_0$ = initial crack length
- $b$ = width of beam
- $d$ = depth of steel reinforcement
- $d_f$ = diameter of fiber.
- $E$ = Young’s modulus of concrete or SFRC
- $e$ = depth of tensile stress block
- $F_{be}$ = bond efficiency factor.
- $F_{td}$ = designed initial tensile strength
\( f_{cm} \) = uniaxial cylinder compressive strength

\( f_{ck,fl} \) = characteristic flexural strength of plain concrete

\( f_{cd} \) = compressive strength

\( f_{fcm,fl} \) = \( f_{ftc,L} \) if bending test is performed; otherwise, \( f_{fcm,fl} \) can be calculated from the compressive strength according to ENV 1992-1-1

\( f_{R,1} \) and \( f_{R,4} \) = residual flexural strength at CMOD = 0.5 mm and 3.5 mm, respectively.

\( f_{sy} \) = yield strength of steel

\( f_{ftc,L} \) = limit proportionality of flexural stress

\( G \) = energy release rate

\( G_c \) = critical energy release rate

\( G_f \) = fracture energy determined by the area under curve of stress-crack width model

\( h \) = total depth of beam

\( h_{sp} \) = notched depth at nonlinear spring hinge

\( J \) = path independent integral around the crack tip

\( K_E \) = kinetic energy

\( K_I \) = stress intensity factor of opening mode I

\( K_{IC} \) = critical stress intensity factor of opening mode I

\( L \) = clear span of a test beam

\( L_f \) = length of fiber

\( L_p \) = length of plastic cracked hinge

\( l_{cs} \) = characteristic length.
$M_n$ = nominal moment capacity
$P_{max}$ = maximum load in bending or tension test
$R_{e,3}$ = equivalent strength ratio at $L/150$ mm according to JCI-SF4 (1984)
$T_{JCI}$ = energy absorbed by the beam up to a load point deflection of $L/150$
$U$ = internal energy
$U_f$ = inelastic strain energy
$U_S$ = surface energy
$U_{es}$ = elastic strain energy
$u_x, u_r$ = deformation in $x$ and $r$ direction, respectively
$V_m$ = matrix volume fraction
$V_f$ = fiber volume fraction
$W$ = external energy
$w$ = crack width
$\delta_d$ = limit (ultimate) design deflection required to the SFRC
$\delta_{limit}$ = deflection limit of $L/150$
$\delta_u$ = deflection corresponding to $P_{max}$
$\varepsilon_1, \varepsilon_2, \varepsilon_3$ = tensile strain points in the RILEM tension model
$\varepsilon'_2$ = transition strain in the post peak tensile stress strain response
$\varepsilon_{fu}$ = ultimate tensile strain
$\varepsilon_{pcr}$ = post crack tensile strain
$\gamma_c$ = partial safety factor for concrete.
\( \gamma_s \) = surface energy of the material

\( \gamma_{xy} \) = shear strain in \( x-y \) coordinate

\( \kappa_h \) = size dependent safety factor

\( \sigma \) = remote stress

\( \sigma_1, \sigma_2, \sigma_3 \) = tensile stress points in the RILEM tension model

\( \sigma^* \) = tensile stress at transition strain in the post peak response

\( \sigma_h \) = equivalent flexural strength according to JCI-SF4 (1984)

\( \sigma_{c,\text{max}} \) = ultimate flexural strength of SFRC

\( \sigma_{cr} \) = first cracking strength of SFRC

\( \sigma_f \) = applied fracture stress

\( \sigma_{m,cr} \) = flexural strength of the unreinforced matrix,

\( \sigma_{pcr} \) = postcrack tensile stress

\( \sigma_t \) = tensile strength in tensile stress block
Fig. 2.1. Fiber pullout model: (a) pull-push test; and (b) pull-pull test
Fig. 2.2. Bond stress versus slip relation (BSR): (a) Naaman et al. 1991, (b) Abrishami and Mitchell 1996 (c) Focacci 2000, (d) theoretical BSR; and (e) Banholzer et al. 2005
Fig. 2.3. Stress strain models for steel fiber reinforced concrete: (a) Lim et al. (1987), (b) Lok and Pei (1998), (c) Lok and Xiao (1999), (d) RILEM TC 162-TDF (2000); and (e) Elasaigh et al. (2004)
Fig. 2.4. Load-deflection curves deduced from flexural or direct tension tests
Fig. 2.5. Tensile stress strain models for SFRC according to Italian guideline
Fig. 2.6. A schematic drawing of an elastic beam with a nonlinear cracked hinge model:

(a) three point beam test; (b) tensile stress strain and crack width model
CHAPTER 3

MODELING OF FIBER-MATRIX INTERFACES

3.1. Introduction

There is a growing interest in the use of fabrics as the main reinforcement in cement based composites. This new class of materials exhibits superior tensile strength and ductility in comparison to other discrete fiber reinforced composites such as steel fiber reinforced concrete (SFRC), glass fiber reinforced concrete (GFRC) and engineering cementitious composite (ECC). The enhanced behavior of fabric reinforced cement composites is primarily governed by interfacial bond characteristics between fabric and matrix. It has been well known that fiber-matrix interface plays an important role in controlling the mechanical performance of cementitious composites. Bond mechanisms of straight, hooked, and crimped fibers using analytical and experimental techniques have been well studied (Kelly and Tyson 1965; Stang et al. 1990; Naaman 1991a,b; Mobasher and Li 1995; Shao et al. 1998). Due to their 2- and 3-D nature, fabrics show a significant improvement over 1-D straight or deformed fibers in the development of mechanical bond. This is partly due to the anchorage of the longitudinal yarns (warp) to the transverse (fill) yarns.

The scanning electron microscopic (SEM) pictures of different types of fabrics structure shown in Fig. 3.1(a)-(c) are used to describe the geometrical and densification effects of the composites. Fig. 3.1(a) presents the mechanical resistance due to non-linear geometry of longitudinal yarns and additional resistance at the cross yarn junctions of the woven fabric. Fig. 3.1(b) demonstrates the uneven pullout load in each fabric layer that induces tortuosity in the path of crack propagation, which increases the energy required
to break the specimen and leads to the enhanced strength and ductility. In addition to the geometrical effect, the effectiveness of fabric embedment in a cement matrix is also important as shown in Fig. 3.1(c). Processing methods such as pultrusion technique or matrix modification using flyash are quite effective means to increase matrix penetration in the interstitial spaces of individual fibers, in between two parallel yarns, and furthermore in the vicinity of junction point of orthogonal yarns. This densification significantly reduces the debonding at the interface transition zone and increases the ability of cross yarn junctions to carry the load, leading to the improved pullout resistance.

Fabric reinforced cement composites have three main characteristics which differentiate them from discrete or uniaxially reinforced fiber composites. The first is the inhomogeneous nature of outer bond between yarn and surrounding matrix and the inner bond among filaments bundled that form a yarn. Mix designs that introduce small pozzolanic particles such as flyash and/or silica fume to pack the micro structure of the cement paste have been shown to result in higher bond strength between yarn and matrix Mobasher et al. (2004) . The processing technique that applies pressure during casting specimens can similarly reduce the capillary porosity and lead to better bonding at interface. Second factor is the slack in longitudinal yarns which is due to the uncrimping nature of individual filament and may also be increased due to shrinkage of paste. The slack in fabric is commonly observed in hand-lay up manufacturing when there is no pre-stretching of the fabric during casting of the samples. The effect of manufacturing techniques to the tensile response of the composite is presented in Fig. 3.2, which shows
that the pre-stretched method yields better tensile performance than the specimen prepared by hand lay-up method. The third aspect is the mechanical anchorage provided at the intersection of longitudinal and transverse yarns, which may have various forms of cross-yarn junctions such as woven, bonded and knitted. Each type may have different load transfer mechanism from longitudinal yarn to the junction and then to the surrounding matrix. The pullout load versus slip response of various fabric structures (woven, warp knitted and bonded) were characterized earlier (Sueki 2003). Results showed resemblance to the pullout response of a straight fiber (Zastrau et al. 2003; Peled and Bentur 2003; Peled et al. 2006), however, the bond strength parameters are quite higher due to the geometrical anchorage mechanisms offered by the transverse yarns. Fig. 3.3 compares pullout responses of a single yarn when embedded as a straight yarn and as a fabric. Note that the response with a fabric configuration is significantly higher.

Interface properties between fabric and cement matrix can be characterized by two main approaches: analytical model and numerical model. For specific bond characteristics between straight fiber and matrix, the closed form solution can be derived explicitly to be used in characterization of that particular composite. In this approach, it assumes that the resistance due to anchorage of transverse yarns is lumped into longitudinal friction in addition to distributed shear resistant at fiber skin. This approach was used to quantify the equivalent bond properties of fabric reinforced cement conducted by Sueki (2003) and Sueki et al. (2007). A series of samples was prepared by various combinations of fabric types, mixture types, embedded fabric lengths, and processing methods. On the other hand, a numerical approach that has more capability to
integrate various mechanical aspects of fabric-matrix interface was also developed in order to better simulate the pullout mechanism of fabric reinforced concrete. The numerical model presented herein can also be used to analyze other composite systems such as reinforced concrete and ferrocement.

3.2. Analytical Fabric Pullout Model

A fabric pullout model was developed based on earlier work by Naaman et al. (1991a,b) with changes to account for the boundary effects. The original model was derived for the test setup with both fiber and specimen in tension. According to the testing arrangement as shown in Fig. 3.4, the fabric is pulled against the bearing plate resulting in the pullout tension force in the fabric that is balanced by compression force in the cement matrix. The original model was changed to account for the difference in force equilibrium.

Fig. 3.5 shows a typical pullout-slip response obtained by pulling the fabric from a section of cement paste. The characteristic of the curve can be divided into three zones based on various stages of shear stress distribution in the fabric. Initially, the curve shows a linear response according to perfect bonding of fabric and cement paste. At a certain point on the ascending curve, the response becomes nonlinear due to initiation of debonding. As the debonding propagates along the embedded length, the stiffness of the load–slip curve decreases along the ascending curve until it reaches the maximum load. The debonding still continues in the postpeak region until the entire length is totally debonded, causing fabric to slide out dynamically. The mechanism of shear stress, force distribution, and derivations for each stage will be explained in the next section.
3.2.1. Mathematical Derivations

As mentioned earlier, fabric embedded in cement matrix can be characterized as a single yarn to obtain equivalent bond properties. Fig. 3.6 presents the schematic drawing of the upper half of a yarn pullout model and the specimen cross section. Static equilibrium requires that, along the embedded length of yarn in matrix, the tensile forces in the yarn $F(x)$ is transferred to the matrix $M$ via the interface and reacted against the restraint plate by normal compressive matrix stresses. This relationship is expressed in the differential form as

$$\frac{dF(x)}{dx} = -\frac{dM(x)}{dx} = \psi \tau(x)$$

(3.1)

where $\psi = \text{equivalent circumference of the yarn}$; and $\tau = \text{shear stress at yarn-matrix interface}$. For small loads, the local shear stress behaves linear elastic as a function of slip with slope defined from the shear strength diagram (Fig. 3.7) and the slip $s$ defined by the difference of the elongation of the yarn $\delta_y$ and shortening of the matrix $\delta_m$

$$\tau = \kappa s = \kappa (\delta_y - \delta_m) = \kappa \int_0^x \left[ \epsilon_y(x) - \epsilon_m(x) \right] dx$$

(3.2)

where $\epsilon_y(x) = F(x)/(A_yE_y)$; $\epsilon_m(x) = -F(x)/(A_mE_m)$; $A$ is cross sectional area; $E$ is Young modulus; and subscripts ‘$y$’ and ‘$m$’ refer to yarn and matrix, respectively. Substituting Eq. (3.2) in Eq. (3.1) and taking the derivative with respect to $x$ yields the differential equation for the yarn pullout force as

$$\frac{d^2F(x)}{dx^2} - \beta^2 F(x) = 0$$

(3.3)
where $\beta^2 = \psi \kappa Q$ and $Q = 1/(A_y E_y) + 1/(A_m E_m)$. The general solution of the second differential equation has the following form

$$F(x) = C_1 e^{\beta x} + C_2 e^{-\beta x} \quad (3.4)$$

The yarn force distribution $F(x)$ is obtained by applying the force boundary conditions $F(0) = 0$ and $F(L) = P$ in Eq. (3.4), which yields

$$F(x) = P \frac{\sinh(\beta x)}{\sinh(\beta L)} \quad (3.5)$$

By taking a derivative of $F(x)$ with respect to $x$, shear stress distribution $\tau(x)$ is obtained

$$\tau(x) = \frac{P \beta \cosh(\beta x)}{\psi \sinh(\beta L)} \quad (3.6)$$

### 3.2.1.1. Pullout Response in Elastic Stages (I)

As long as the shear stress at interface is less than the maximum shear strength $\tau_{\text{max}}$, yarn and matrix are fully bonded as shown by dashed line in Fig. 3.8(a) and the applied load in Stage I is less than the maximum bonded load ($P_1 < P_{1b,\text{max}}$). The slip at the end of the yarn is obtained by integrating slip response up to $x = L$

$$s(L) = \int_0^L \left[ \frac{F(x)}{A_y E_y} + \frac{F(x)}{A_m E_m} \right] dx = \frac{PQ}{\beta \sinh(\beta L)} \left[ \cosh(\beta L) - 1 \right] \quad (3.7)$$

When shear stress at $x = L$ reaches the maximum strength $\tau_{\text{max}}$, the elastic response is terminated and the maximum bonded load $P_{1b,\text{max}}$ is calculated by

$$P_{1b,\text{max}} = \frac{\tau_{\text{max}} \psi}{\beta} \tanh(\beta L) \quad (3.8)$$
3.2.1.2. Pullout Response in Nonlinear Stage (II)

Loading beyond the elastic limit is initiated when the shear stress exceeds the maximum shear strength $\tau_{\text{max}}$. This leads to debonding for a length of $d$ on the right of Fig. 3.8(b) while the two materials are still perfectly bonded on the left portion ($L-d$). The shear stress distribution is governed by a constant frictional stress $\tau_{\text{frc}}$ along the debonded zone in addition to a shear lag model terminating with the maximum bond strength value $\tau_{\text{max}}$ at the debonding junction. The algorithm for the modeling is based on incrementally increasing the debonded length and calculating the applied force from the shear stress distribution. In stage II, the resistant to pullout load $P_2$ is calculated by the summation of the two forces: $P_{2b,\text{max}}$ and $P_d$ in the bonded and debonded regions

$$P_2 = P_d + P_{2b,\text{max}} = \tau_{\text{frc}} d + \frac{\tau_{\text{max}}}{\beta} \tanh(\beta(L-d))$$  \hspace{1cm} (3.9)$$

$$P_{2b,\text{max}} = \frac{\tau_{\text{max}}}{\beta} \tanh(\beta(L-d)).$$  \hspace{1cm} (3.10)$$

The force boundary conditions for nonlinear stage are slightly modified $F(0) = 0$, $F(L-d) = P_{2b,\text{max}}$, and $F(L) = P_2$ according to the bonded and debonded zones. Apply these boundary conditions in Eq. (3.4), one can obtain force distributions $F_b(x)$ and $F_d(x)$ in bonded and debonded zones

$$F_b(x) = P_{2b,\text{max}} \frac{\sinh(\beta x)}{\sinh(\beta(L-d))}, \quad 0 \leq x \leq L-d$$

$$F_d(x) = P_{2b,\text{max}} + \tau_{\text{frc}} \psi (x - L + d), \quad L-d \leq x \leq L$$  \hspace{1cm} (3.11)$$

Slip at the end of the yarn is obtained in a same way as Eq. (3.7) in Stage I but carrying out for two separate regions, bonded and debonded.
\[
\begin{align*}
    s(L)_2 &= \int_0^{L-d} \left[ \frac{F_h(x)}{A_y E_y} + \frac{F_b(x)}{A_m E_m} \right] dx + \int_{L-d}^L \left[ \frac{F_d(x)}{A_y E_y} + \frac{F_d(x)}{A_m E_m} \right] dx \\
    &= \frac{P_{2b, \text{max}} Q \left[ \cosh(\beta(L-d)) - 1 \right]}{\beta \sinh(\beta(L-d))} + \frac{1}{2} Qd \left( \tau_{frc} \psi d + 2P_{2b, \text{max}} \right)
\end{align*}
\]

(3.12)

3.2.1.3. Pullout Response in Dynamic Stage (III)

Stage III is defined as a dynamic response [Fig. 3.8.(c)], which consists of two sub stages: initial and rigid sliding. At the initial stage, it is assumed that the yarn is completely debonded but not yet sliding \( (\Delta d = 0) \). The shear stress still remains at frictional shear resistance \( \tau_{frc} \) as shown in Fig. 3.8.(c.1), resulting in the pullout load and slip of

\[
P_{3,1lt} = \tau_{frc} \psi L
\]

(3.13)

\[
s(L)_{3,1lt} = \int_0^L \left[ \frac{F_d(x)}{A_y E_y} + \frac{F_d(x)}{A_m E_m} \right] dx = \frac{1}{2} Q\tau_{frc} \psi L^2
\]

(3.14)

where \( F_d(x) = \tau_{frc} \psi x \) for \( 0 < x < L \). As the load continues to decrease the slip continues to increase. When the load drops below a certain level, the slip begins to decrease (snap back) due to material unloading. Snap back response is not possible in the displacement control because the end slip is incrementally imposed by the pulling grip. Thus, the second phase of dynamic sliding \( (\Delta d > 0) \) is assumed to take place when the snap back starts. The resisting shear stress is assumed to drop to the dynamic shear strength \( \tau_{dyn} \) leading to a simplified rigid body motion. The embedded length is reduced to \( (L-\Delta d) \), and the dynamic pullout force at the end is

\[
P_{3,uni} = \tau_{dyn} \psi (L-\Delta d)
\]

(3.15)
From Fig. 3.8.(c.2) and force equilibrium, one can write force distribution as

\[ F(x) = P_{3,n} + \tau_{dyn} \psi(x - L + \Delta_d), \quad 0 \leq x \leq L - \Delta_d \]  \hspace{1cm} (3.16)

The slip at the end of the yarn is also calculated in the same way as Eq. (3.14), except that the embedded length is shorter.

\[ s(L)_{3,n} = \int_0^{L - \Delta_d} \left[ \frac{F(x)}{A_y E_y} + \frac{F(x)}{A_m E_m} \right] dx = \frac{\tau_d \psi Q}{2} (L - \Delta_d)^2 + P_{3,n} Q (L - \Delta_d) \]  \hspace{1cm} (3.17)

The total end slip of the yarn measured in dynamic stage comes from the slip at the last step of Stage II (static) \( s(L)_{2, last} \) and the dynamic slip in Stage III

\[ s_{total}(L)_{3,1st} = s(L)_{2, last} + s(L)_{3,1st} \]  \hspace{1cm} for initial \( \Delta_d = 0 \), and
\[ s_{total}(L)_{3,n} = s(L)_{2, last} + s(L)_{3,n} \]  \hspace{1cm} for rigid sliding \( \Delta_d > 0 \) \hspace{1cm} (3.18)

It should be noted that the measured sliding displacement in Stage III \( D_{measured}(L) \) from the experiment consists of the total slip given by Eq. (3.18) and the rigid body displacement \( \Delta_d \)

\[ D_{measured}(L)_{3,n} = s(L)_{2, last} + s(L)_{3,n} + \Delta_d \]  \hspace{1cm} for rigid body \( \Delta_d > 0 \) \hspace{1cm} (3.19)

\textbf{3.2.2. Algorithms for Pullout Simulations}

The pullout response obtained from the experiment can be simulated using the analytical model described in the preceding section. The simulation procedure can be summarized as follows:

1. Average the experimental pullout responses of \( N \) samples at every slip level to get a representative curve;
2. In elastic Stage I, assume a pullout load $P_1$ and calculate slip $s(L)$ by Eq. (3.7). Increase the value of $P_1$ until it reaches $P_{1b,max}$ defined by Eq.(3.8), which is the termination point of Stage I;

3. In nonlinear Stage II, incrementally impose the debonded length $d$ and calculate the corresponding load $P_2$ and slip $s(L)$ from Eqs. (3.9) and (3.12). It is assumed Stage II ends and the dynamic mode begins when the computed slip $s(L)$ start to decrease (snap back); and

4. In dynamic Stage III, the first measured response ($P_{3,1st}$, $s_{total}(L)_{3,1st}$), in which no sliding occurs ($\Delta d > 0$), can be determined by Eqs. (3.12)-(3.14), and (3.18). When the yarn begins to slide out dynamically ($\Delta d > 0$), the load and total slip ($P_{3,nth}$, $s_{total}(L)_{3,nth}$) are calculated by Eqs. (3.12), (3.15), (3.17) and (3.18). Finally, the measured sliding displacement at the end of the fiber defined by Eq. (3.19) is used in simulation.

3.2.3. Pullout Experiments

3.2.3.1. Mix Design

Two different formulations for the matrix were used in the fabric pullout tests as presented in Table 3.1. The control matrix consisted of a Portland cement paste with 5% by volume of silica fume. To enhance the workability of the mix, 0.1% by volume of superplasticizer was added. Another mixture with 40% by volume of fly ash as replacement to cement was prepared to study the effect of fly ash on fabrication and bonding.
3.2.3.2. Fabric Types

Table 3.2 presents four fabric types used in the study. They were different in modulus and textures; the stiffness ranges from the high modulus alkali-resistance glass (AR-glass) and polyvinyl alcohol (PVA) fabrics to the low modulus polypropylene (PP) and polyethylene (PE) fabrics. The textures of these fabrics are as follows: bonded AR-glass [Fig. 3.9(a)], warp knitted weft insertion PP [Fig. 3.9.(b)] and woven PE and PVA [Fig. 3.9(c)]. In bonded fabrics a perpendicular set of yarns (warp and weft) are attached together at the junction points using a polymeric adhesive. In knitted fabrics the yarns in the warp direction are knitted into stitches to assemble with straight yarns used in the weft direction. In these two fabrics the reinforcing yarns in the composite are in a straight form. In woven fabrics the warp and the fill (weft) yarns pass over and under each other and the reinforcing yarns are in a crimped shape. The modulus of elasticity of the yarns in a fabric, as well as the geometry of the reinforcing yarns and the nature of the junctions of the fabrics, can all contribute to the pullout behavior and bonding of the various systems.

The bonded AR glass fabric contains 4 yarns/cm in each direction of the fabric. The yarns were impregnated with a polymeric coating. The multifilament PP fabric was weft inserted knitted and made from 0.8 yarns/cm in the reinforcing direction (warp yarns) and 8 yarns in a perpendicular direction (weft yarns). The woven fabrics are made either from monofilament PE or multifilament PVA. The density of the yarns in the woven PE is 22 yarns/cm in the reinforcing direction (weft yarns) and 6 yarns in a perpendicular direction (weft yarns). For PVA fabric, there are 5.5 yarns/cm in each direction.
3.2.3.3. Sample Preparation

All specimens were made in a single mold with dimensions of 254 x 152.4 mm as shown in Fig. 3.10. The paste was poured into the mold in two layers and a “clean” fabric was placed in between them. This procedure was done for all fabric types and is referred to as the cast process. For the AR-glass, PP, and PVA, another set of specimens was prepared using the pultrusion technique (Peled and Mobasher 2003). In this process a layer of fabric was first passed through a cement bath, and then pulled through a set of rollers to squeeze the paste into the openings of the fabric while removing excessive paste. The impregnated fabric was then laid between two layers of cement paste similar to the cast process. Another process was examined using vacuum processing of the paste prior to specimen preparation, which applies to AR-glass, PP, and PVA fabrics. After the ingredients were blended in a mixer, the fresh mixture was transferred to a sealed container and additional mixing was conducted under vacuum for 2 minutes. A “clean” single layer of fabric was then laid between two layers of the vacuumed paste.

For all processing methods the fabrics were held under a control tensile load of 4.25 N per 25 mm width. The tension was released after the specimen was cast; the specimens were demolded 24 h after casting, and transferred to an oven at 80°C with a relative humidity of 100%. Two days after steam curing, the samples were cut to the required embedded length using a water cooled saw with a diamond edge blade as shown in Fig. 3.11. Then they were stored at room temperature until testing at 7 days. Each specimen was 8.1 mm thick and 25.4 mm wide with its length equal to the embedded length of the fabric at either 12.7 or 7.6 mm.
Specimens prepared for 15 test series are presented in Table 3.3. They were labeled using five components: mix types, control (-) and fly ash (F); fabric types, AR-glass (G), polypropylene (PP), polyethylene (PE), and polyvinyl alcohol (PVA); processing methods, cast (-), pultrusion (P) and vacuum (V); followed by free length and embedded length (in inches). For example, Sample G105 refers to a specimen using control mix, AR-glass fabric (G), made by the cast process, with 1 in. free length of fabric and 0.5 in. embedded length, while Sample FGP105 refers to a specimen using fly ash mix (F), AR-glass fabric (G), made by the pultrusion process (P), with 1 in. free length of fabric and 0.5 in. embedded length.

### 3.2.3.4. Pullout Tests

Pullout tests were carried out using an Instron testing machine at a crosshead rate of 0.25 mm/s. The experiment was designed to test five samples for each series; however, during the test some samples failed during installation or exhibited yarn failure prior to test completion, additional samples were prepared and tested to make up the potential loss of data. The total samples for each series vary from four to eight, however only four samples per batch were selected to calculate the average and standard deviations. Samples with erratic behavior due to gripping misalignment or testing failure were removed as outliers. Unfortunately, Series FPVAP103 had only three samples (Table 3.4).

The deformation of the free fabric length between the edge of the sample and the machine grip, as previously shown in Fig. 3.4, affects the slip deformation, and its magnitude should be accounted for in the measurement of interface properties (Shao et al.
In order to account for the free fabric length, three samples for each plain fabric with the size of 53x165 mm were tested in tension and values of apparent Young modulus of yarn $E_y$ are reported in Table 3.3. The modulus was used to estimate the elongation of the free fabric length ($L_{FreeLength}$) due to pullout force. Adjustment to the measured slip data included the correction to slip in the free fabric length in addition to the linearization of the initial loading stage during which the stretching of the crimp and alignment of the fabric takes place.

Fig. 3.12 presents pullout curves of the control mix with different fabric types prior to and after adjustment of the free length and initial deformation. The pulled out yarns of glass fabrics and PP are straight [Fig. 3.9.(a) and (b)] and therefore only slightly affected by the free length. On the other hand, the PE fabric is most affected by the free length due to its low axial stiffness $A_yE_y$ (Table 3.3) and woven structure, i.e., crimped shape of the pulled yarns [Fig. 3.9(c)]. PVA has also a woven structure but shows less sensitivity.

The adjusted load-deformations are tabulated in Table 3.4 and represent the initial stiffness of the load-slip curve, maximum load, slip at maximum load, toughness (area under the pullout response up to the maximum load), and nominal shear strength $\tau_{nom}$, defined as

$$\tau_{nom} = \frac{P_{max}}{N\pi\phi L}$$  \hspace{1cm} (3.20)

where $P_{max} =$ maximum load; $N =$ number of yarns pulled out; $\phi =$ effective diameter of a yarn; and $L =$ embedded length of fabric.
The experimental pullout slip responses of replicate samples were used in order to obtain a mean representative average curve for each test series. Using spline interpolation techniques, the load response in each tested specimen was sampled at multiple slip levels. The average of the pullout load at that level was used as the mean representative curve. Fig. 3.12 shows an example of the data analysis to obtain a mean representative curve for the control test series and Fig. 3.13 shows the mean representative curves for all 15 test series. For PP105 series, in addition to the mean representative curve, two additional curves were also generated representing the upper and lower bound estimates of the curve based on a single standard deviation of the representative curves ($\mu \pm 1\sigma$). The coordinates of these curves were obtained by calculating $\mu \pm 1\sigma$ for each set of responses. The representative curves of the same fabric type are presented together including the different experimental parameters (various mixes, embedded lengths, and processing methods). Table 3.4 verifies that the key parameters (initial stiffness, maximum load, slip at maximum load, toughness, and nominal shear strength) evaluated from one representative curve, compare well with the average of four samples.

3.2.4. Simulations of Pullout Tests

The response of each test series was simulated using material properties of yarn and cement pastes provided in Table 3.3 as input parameters. Internal interface parameters ($\tau_{max}$, $\tau_{frc}$, $\tau_{dyn}$ and stiffness efficiency factor $\eta$) are estimated in order to find a match between the simulation and experiment. The stiffness efficiency factor parameter $\eta$ is defined based on the observation that a nominal estimation of the stiffness of a yarn ($A_yE_y$) overestimates the experimental values significantly. This is due to several factors
including the variations in bond of the interface zone, inefficiency due to the sleeve effect, lack of uniform strain in all the yarns at the beginning and during the test, the initial curvature in the yarns, and lack of bonding of all the filaments in the yarn due to imperfections and porosity. To obtain a realistic fit to the experimental data using material properties of the components, the stiffness efficiency factor ($\eta < 1$) was introduced as a reduction to the nominal fabric stiffness to account for the inefficiencies in the bonding. This parameter represents the percentage of effective filaments, which actively contribute to the apparent axial stiffness ($\eta A_x E_y$) of the yarn. The efficiency factor has also been independently verified by observations indicating that sleeve filaments bonded to the matrix contribute to axial stiffness while the core filaments provide marginal stiffness when unbonded multifilament yarns are used (Banholzer 2004).

The averaged responses and their simulations of the control test series (cast without fly ash G105, PP105, PE105, and PVA105) are presented in Fig. 3.14. The simulation clearly demonstrates the fit between the model and the experiments. Note that the abrupt changes in the late postpeak responses are due to the sudden change of shear strength from $\tau_{frc}$ to $\tau_{dyn}$. The remaining representative curves were also conducted in the same way to determine the internal parameters and reported in Table 3.4 along with the key parameters obtained from experiments.

To estimate the shear strength parameters of all the pullout data with their distribution (including their standard deviation), the ranges of shear strength parameters were determined by fitting the simulation responses to the upper and lower bound of
experimental responses defined by the $\mu \pm 1 \sigma$ curve. Test series PP105 in Fig. 3.15 was selected to demonstrate this procedure and the range of internal shear strength parameters obtained by this approach (from the best fit) is reported in Table 3.5.

### 3.2.5. Results and Discussions

The simulation described above is used to identify the internal shear parameters (Table 3.4). Fig. 3.16(a) compares the nominal shear strength $\tau_{\text{nom}}$ obtained from experiments, for all tested series, and the internal shear strength parameters $\tau_{\text{max}}$ and $\tau_{\text{frc}}$ obtained from the best fit of the simulation curves. It should be noted that the pullout of the fabric was modeled by an equivalent yarn (ignore transverse yarns in the model). The fitted value of dynamic frictional strength $\tau_{\text{dyn}}$ may be higher than the frictional strength $\tau_{\text{frc}}$, which occurred in some glass series 1–3 in Table 3.4. This increase could happen if the resisting force of the transverse yarns plays a major role after the bond failure of the longitudinal yarn. Nominal shear strength $\tau_{\text{nom}}$, which is the average strength at maximum load, lies between $\tau_{\text{max}}$ and $\tau_{\text{frc}}$ as expected. The correlation of these shear strength parameters is shown in Fig. 3.17. Note that as the nominal shear strength increases, both the maximum and frictional shear strength values increase as well for a range of specimens tested. Since nominal shear strength agrees well with the internal shear strength parameters, this single quantity can be used as a measurement to determine the bond strength characteristic of the fiber-matrix system. The effects of the various tested parameters such as fabric types, matrix types, embedded lengths, and processing methods on the shear strength parameters were discussed in detail in Peled et al. (2006).
In general, the greatest bond is reported for the glass fabric, whereas the PE and PVA show the lowest bond strengths [for the control test series G105, PE105, and PVA103 Fig. 3.16(b)]. Good bonding of the glass fabric is expected due to its high modulus of elasticity (78 GPa, Table 3.2). The low bond of the PVA might be attributed to the fact that the diameter of the bundle is relatively large with increased number of filaments (Tables 3.2), leading to dense fabric structure and poor penetration of the cement between the yarns and the filaments as observed by the scanning electron microscope (SEM), resulting in poor bonding. Addition of fly ash improved bonding by 35% in the case of the glass fabric but slightly decreases bonding -16% in PVA fabric (G105 vs. FG105 and PVAP103 vs. FPVAP103 [Fig. 3.16(b)]). In the glass system the improved bonding might be attributed to improved durability of the glass fabric when fly ash is used. The difference of embedded length (7.6 and 12.5 mm) does not significantly influence the shear strength values, for PP (PP103 vs. PP105 and PPP103 vs. PP105) as well as for PVA fabrics (FPVAP103 vs. FPVAP105 [Fig. 3.16(b)]). The pultrusion technique increases bond strength in PP roughly by 72% (PP105 vs. PPP105) and PVA by 85% (PVA103 vs. PVAP103) but it slightly decreases strength in the AR-glass by 10% (G105 vs. GP105). The improvement in bonding by the pultrusion method of the PP system and PVA is explained by better penetration of the cement matrix between the filaments of the bundle, as discussed in detail in Peled et al. (2004). In contrast, the vacuum technique indicates an increase in bond strength in AR-glass by 35% (G105 vs. GV105), but decrease in PP system by 38% (PP103 vs. PPV103). The higher viscosity and poor penetrability of vacuumed paste within the fine grid and small filament spacing of PP
fabric may explain this decrease. In comparison, the coarse grid AR-glass fabric made from resin-coated bundles [Fig. 3.9(d)] performs much better with the vacuum paste.

It is found that factor $\eta$ representing the efficiency of the fabric in consideration of the stiffness of a single yarn for all 15 simulations shown in Table 3.4 is less than 1. The efficiency of yarn obtained from the control series ranked from highest to lowest is PE105 (24.0%), PP105 (17.0%), PVA103 (15.5%), and G105 (5.0%). The results are directly correlated with the number of yarns embedded in the matrix for various fabric systems: PE, PP, PVA, and glass with 39, 23, 8, and 8 yarns, respectively. This observation indicates that as the number of yarns carrying the load increases (more fabric area), a higher efficiency factor $\eta$ is obtained. Note that the efficiency of the PVA is only slightly lower than that of the PP; however, the number of filaments in the PVA bundle is relatively high (Table 3.2), increasing the fabric area. It is also noticed that the use of the pultrusion technique with fly ash in PVA tremendously increases the area efficiency; FPVAP103 ($\eta = 32\%$) and FPVAP105 ($\eta = 85\%$), (Table 3.4). This may be due to the nature of the yarns (multifilament), the impregnation processing method (pultrusion), and the low viscosity properties of the matrix due to the presence of the fly ash particles, which combine to allow the cementitious matrix to penetrate between the filaments of the bundle such that more fibers can contribute to axial stiffness.

3.3. Numerical Fabric Pullout Model

In order to better characterize bond properties between fabrics and cement matrix, the effect of mechanical anchorage due to transverse yarn should be taken into account. In addition the shear strength diagram used in the derivation of analytical fabric pullout
model presented in section 3.2 is not flexible to be used in various composite systems that may require different shapes of shear strength diagram. Other aspects such as slack found in composites made by hand lay up process shall be included in the model. When several features are desired to be included in the pullout model, the derivation to obtain analytical form becomes extremely tedious or even not possible. In this case, the numerical approach that treats mechanical aspects into sub models and integrates them to a form a set of algebraic equations provides a viable solution in modeling of complex fabric-matrix interfaces. Finite difference scheme was selected in this study due to its simplicity yet capable of integrating all necessary aspects of fabric reinforced cement composites.

3.3.1. Free Form Nonlinear Material Models

Constitutive material behaviors are described by piecewise linear functions as depicted in Fig. 3.18. Fig. 3.18(a) represents nonlinear bond slip model that shear stress $\tau$ is a function of slip $s$. The secant modulus $k$ enforces the material constitutive behavior of local shear stress and slip at each node of the finite difference model. Fig. 3.18(b) presents a general longitudinal yarn stress strain model that may begin with initial slack, which simulates the delay in load-deformation response of the yarn. Similar to the bond slip model, secant modulus $E_y$ is used to ensure stress strain relationship of the longitudinal yarn in a finite difference model. Finally, a nonlinear spring model as shown in Fig. 3.18(c) represents the additional stiffness provided at cross yarn junctions. This feature is only active at the junction points, representing the geometrical feature of the fabric and depending on the weave density. With similarity of using multi-linear-segments to describe material models and using secant modulus to control material
constitutive behaviors, all material models can be implemented as a unified linear piecewise function in the finite difference model.

### 3.3.2. Derivation of Finite Difference Fabric Pullout Model

Finite difference framework was employed to integrate mechanical aspects of fabric-matrix interface, longitudinal yarn and cross yarn junction to represent a numerical fabric pullout model. Nodal equilibrium equations can be derived from free body diagram of typical nodes in the finite difference model. Material constitutive behaviors are ensured by a material stiffness updating algorithm.

#### 3.3.2.1. Finite Difference Equilibrium Equations

In formulating the nodal equilibrium equations, a variable slip $s$ is defined as the relative difference between the elongation of the longitudinal yarn and cement matrix.

$$s = \int (\varepsilon_y - \varepsilon_m) \, dx$$  \hspace{1cm} (3.21)

where $\varepsilon_y$ and $\varepsilon_m$ are yarn and matrix strain, respectively. For fabric reinforced cement composites with a relatively low fiber volume fraction (< 4%), the axial stiffness of the yarn $A_y E_y$ is significantly lower than the axial stiffness of the matrix $A_m E_m$; thus, the effect of matrix strain can be ignored and the slip is simplified to

$$s = \int \varepsilon_y \, dx \quad \text{and} \quad s' = \frac{ds}{dx} = \varepsilon_y$$  \hspace{1cm} (3.22)

where $dx$ is a finite length.

Fig. 3.19(a) presents a finite difference model of a fabric pullout specimen previously shown in Fig. 3.18(d). The embedded length $L$ is discretized into “$n$” nodes with equal spacing of $h$. The bond stress is assumed constant over the small spacing $h$ for each node.
within the linear domain. Two types of nodes are used; nodes along the length of the yarn which receive contribution from the interface bond forces, and nodes at each junction point which connects to an additional spring simulating the transverse yarns. Two boundary conditions are imposed. At the left end, node 1, force in longitudinal yarn is imposed to be zero, implying that the yarn strain or derivative of slip vanishes \((\varepsilon_{y1} = s'_1 = 0)\). At the right end, node \(n\), the nodal slip is prescribed incrementally, simulating displacement control. As the loading progresses, the part of the longitudinal yarn that slips out of the matrix has no frictional bond resistance; thus, yarn elongation is the only term in that section. For \(n\) number of nodes used in the finite difference model, only three typical nodes labeled A, B and C are necessary to derive for nodal equilibrium equations; other nodes are replicates of these nodes.

### 3.3.2.1.1. Boundary Condition at the Left End

Fig. 3.19(b) shows a free body diagram of a force boundary condition at the typical node “A”. The force in longitudinal yarn at distance \(+h/2\) from the first node 1, is defined as \(F_{1+1/2}\) and must be balanced with the bond resistance force over the first node 1, \(B_1\).

\[
F_{1+1/2} - B_1 = 0 \tag{3.23}
\]

\(F_{1+1/2}\) is calculated by using the yarn area \(A_y\), average secant modulus \(E_y\) at nodes 1 and 2 and the yarn strain at node \(I\), which is approximated by forward difference method:

\[
F_{1+1/2} = A_y \left( E_{y,1} + E_{y,2} \right) \frac{(s_2 - s_1)}{h} \tag{3.24}
\]
Bond stress at node \( i \) is determined by the secant modulus \( k_i \) and slip \( s_i \). The resistance bond force \( B_i \) is then obtained by integrating the bond stress on the surface area of the yarn defined by the perimeter \( \psi \) over a half spacing \( h/2 \).

\[
B_i = \frac{1}{2} \psi k_i s_i h
\]  
(3.25)

By substituting Eqs. (3.24) and (3.25) into Eq. (3.23) and rearranging terms, the equilibrium of force for the typical node “A” can be expressed as

\[
- \left[ A_y \left( \frac{E_{y,i} + E_{y,2}}{2} \right) + \frac{1}{2} \psi k_i h^2 \right] s_1 + A_y \left( \frac{E_{y,1} + E_{y,2}}{2} \right) s_2 = 0
\]  
(3.26)

### 3.3.2.1.2. Interior Nodes

Figs. 3.19(c and d) show two typical nodes “B”, and “C” which describe two possible types of interior node \( i \); those connected and those not connected to the cross yarn junctions. The force equilibrium for these two types of nodes can be represented by the following equations

\[
F_{i+1/2} - F_{i-1/2} - B_i - G_i = 0
\]  
(3.27)

\[
F_{i+1/2} - F_{i-1/2} - B_i = 0
\]  
(3.28)

The forces at the half right (+h/2) and half left (-h/2) of the node \( i \) are calculated by:

\[
F_{i+1/2} = A_y \left( \frac{E_{y,i} + E_{y,i+1}}{2} \right) \frac{(s_{i+1} - s_i)}{h}
\]  
(3.29)

\[
F_{i-1/2} = A_y \left( \frac{E_{y,i-1} + E_{y,i}}{2} \right) \frac{(s_i - s_{i-1})}{h}
\]  
(3.30)
The bond force at node $i$ is calculated from the distributed bond stress over a spacing of $h$:

$$B_i = \psi k_i s_i h$$  \hspace{1cm} (3.31)

The spring force at node $i$ can be expressed as a function of slip $s_i$ and the secant modulus of the material model $g_i$.

$$G_i = g_i s_i$$  \hspace{1cm} (3.32)

By substituting Eqs. (3.29)-(3.32) in Eqs. (3.27) and (3.28), two equilibrium equations for interior nodes with and without spring are obtained:

$$Ay \left( \frac{E_{y,i-1} + E_{y,i}}{2} \right) s_{i-1} - \left[ Ay \left( \frac{E_{y,i-1} + 2E_{y,i} + E_{y,i+1}}{2} \right) + \psi k_i h^2 + g_i h \right] s_i + Ay \left( \frac{E_{y,i} + E_{y,i+1}}{2} \right) s_{i+1} = 0$$  \hspace{1cm} (3.33)

$$Ay \left( \frac{E_{y,i-1} + E_{y,i}}{2} \right) s_{i-1} - \left[ Ay \left( \frac{E_{y,i-1} + 2E_{y,i} + E_{y,i+1}}{2} \right) + \psi k_i h^2 \right] s_i + Ay \left( \frac{E_{y,i} + E_{y,i+1}}{2} \right) s_{i+1} = 0$$  \hspace{1cm} (3.34)

3.3.2.1.3. Boundary Condition at the Right End

In simulation of displacement control, the slip $s^*$ at the right end is incrementally imposed for each load step until it reaches a prescribed value. Since this slip is known in advanced, no equilibrium equation for the end node $n$ is required; but, the slip will be implemented in the equilibrium equation of node $n-1$. This can be accomplished by replacing the node index $i-1, i, i+1$ with $n-2, n-1, n$ and the slip $s_n$ with $s^*$ in Eq. (3.33) or (3.34). The next step is to rearrange the unknown terms to the left and moving the known
terms to the right as the driving force. This yields two equations for the node \( n-1 \) that are connected and not connected to spring:

\[
A_y \frac{E_{y,n-2} + E_{y,n-1}}{2} s_{n-2} - \left[ A_y \frac{E_{y,n-2} + 2E_{y,n-1} + E_{y,n}}{2} + \psi k_{n-1} h^2 + g_{n-1} h \right] s_{n-1}
= -A_y \frac{E_{y,n-1} + E_{y,n}}{2} s^* \\
A_y \frac{E_{y,n-2} + E_{y,n-1}}{2} s_{n-2} - \left[ A_y \frac{E_{y,n-2} + 2E_{y,n-1} + E_{y,n}}{2} + \psi k_{n-1} h^2 \right] s_{n-1}
= -A_y \frac{E_{y,n-1} + E_{y,n}}{2} s^* 
\tag{3.35} \tag{3.36}
\]

3.3.2.2. Matrix Assemblage

The 1-D pullout model is obtained by discritizing an embedded length \( L \) into \( n \) nodes and applying appropriate nodal equilibrium equations expressed in Eqs. (3.26), (3.33) - (3.36) for each type of node from node 1 to \( n-1 \) and the assemblage of the nodal equations in the following form

\[
[C]_{(n-1),(n-1)} \{S\}_{(n-1)} = \{P\}_{(n-1)} \tag{3.37}
\]

where \([C]\) is a coefficient matrix containing coefficient terms. \( \{S\} = \{s_1, s_2, s_3, ..., s_{n-1}\} \) is an unknown nodal slip vector and \( \{P\} = \{0, 0, 0, ..., -A_y(E_{y,n-1} + E_{y,n})s^*/2\} \) is the known driving force vector. Note that the coefficient matrix \([C]\) is a tridiagonal matrix containing a band of three non zero terms; thus, this system of equation can be solved efficiently by Thomas’s algorithm (Olver and Shakiban 2006).
3.3.2.3. Material Stiffness Updating and Convergence

The algorithm for updating material stiffness and checking for convergence are presented as a flow chart in Fig. 3.20. For each step of imposed slip, the system of equations is solved for nodal slip values. According to Eq. (3.22), the yarn strain at each node can be computed using forward, central, and backward finite difference method for node 1, 2 to \( n-1 \) and \( n \), respectively. Then the slip and yarn strain are used to calculate the distribution of bond stress, yarn tensile stress, and spring forces according to their material models. Next, the secant modulus \( k, E_y \) and \( g \) are calculated by dividing the current forces with deformations.

In order to satisfy the convergence criterion, all updated secant modulus values must be sufficiently close to their values in the previous iteration. To calculate the relative change of secant modulus between the two iterations, the steepest positive slopes of material models are used as references \( k_{ref}, E_{yref} \) and \( g_{ref} \). If all the relative changes are less than a specified tolerance level at all nodes, the constitutive laws are satisfied for the whole domain. The next slip increment can be imposed and the process continues until the total slip reaches the specified final value. If the secant modulus at some nodes does not converge, the next secant modulus \( k^{(j+1)}, E_y^{(j+1)} \) and \( g^{(j+1)} \) will be estimated from the previous values \( k^{(j-1)}, E_y^{(j-1)} \) and \( g^{(j-1)} \) and the current values \( k^{(j)}, E_y^{(j)} \) and \( g^{(j)} \) by the weighting factors \( \xi_k, \xi_{Ey} \) and \( \xi_g \), which vary between 0 and 1. Small weighting factor gradually change the next secant modulus, leading to stable process but slower convergence while the larger values lead to faster convergence but less stable. For highly nonlinear problems, the rapid changes of modulus between two iterations may result in
non convergence at all. The default values for weighting factor between 0.2 - 0.3 were found to be efficient for most moderate to severe nonlinear problems.

### 3.3.3. Parametric Studies of Material Parameters

Parametric studies are conducted to address three main characteristics of fabric reinforced cement composites: bond-slip relationship, slack in a yarn, and mechanical anchorage provided at the cross yarn junctions. To demonstrate these effects, a numerical fabric pullout specimen as shown in Fig. 3.21 was used with benchmark mechanical and geometrical parameters as follows: the yarn modulus of 5000 MPa, the longitudinal yarn diameter of 0.5 mm, an embedded length of 12 mm. For the case study of the cross yarn junctions, a weave density of one yarn per 6 mm were assumed; thus, two springs are placed symmetrically at nodal locations 3 and 9 mm from the left end. Fig. 3.22(a)-(c) shows three material models and Table 3.6 shows different combinations of material models used in parametric studies. The first row shows a base numerical model, which consists of a base bond model \( \tau_{\text{max_base}} = 2 \text{ MPa} \) and a yarn stress strain model without slack. Subsequent models below show other combinations, in which the base numerical model is modified; the bond strength may be increased; the yarn model may be added with different degree of slack; or the longitudinal yarn may be connected to springs at different degree of restraining. In brief notations, “\_base” refers to a basic material model needed to be included in a base numerical model while “\_opt” refers to optional parameters or models that can be added to the base numerical model. The magnitude of base/opt models can be amplified by a scaling factor.
3.3.3.1. Effect of Slack and Cross Yarn Junctions to the Pullout Responses

The first set of parametric study compares the pullout responses of the base numerical model with its two modified models: including slack (+1 x $\varepsilon_{\text{slack, opt}}$) and including springs (+1 x $SF_{\text{max, opt}}$). Figs. 3.23(a and b) show the distribution of slip, bond stress and yarn force along the embedded length of the fabric at two stages: the slip at the right end reaches 0.5 mm and 1.0 mm. These two slip values correspond to the two controlling points: peak stress and postpeak specified in the base bond model (1.00 x $\tau_{\text{max, base}}$) [Fig. 3.22(a)].

At the bottom of Fig. 3.23(a), the slips at the right ends of the three models are all equal to 0.5 mm as prescribed in the displacement control. When compared to the base numerical model, it can be seen that the effect of slack in longitudinal yarn substantially decreases the slip distribution while the spring marginally decreases the distribution. The above subplot shows the corresponding bond stress. At the right end, bond stresses on the extruding parts of distance 0.5 mm are zero. The bond stresses next to the left, corresponding to slips between 0.1 and 0.5 mm, vary in narrow range of 1.9 and 2.0 MPa as specified by the base bond model. In the left end region, the effect of slack considerably decreases the bond stress as the slip is very low while the effect of spring is negligible. The top subplot shows the yarn force distribution, which can be noticed that, the effect of slack decreases the force distribution especially in the left half due to lower bond stress. The effect of spring is observed at two nodal spring locations 3 and 6 mm, which add more resistant force in addition to the bond stress, resulting higher pullout force at the right end.
At the bottom of Fig. 3.23(b), the slip at the right end reach 1.0 mm in the postpeak of base bond model. The trend of slip distribution remains the same that the effect of slack yields the least slip while the spring yields a lot more and closes to that of the base numerical model. In the middle subplot, it reveals that the bond stress distribution of the base numerical model and the modified model including springs are very close to one another but significantly lower than the modified model including slack. This due to the slip distributions of the first two model are in the postpeak region (slip > 0.5 mm) while the slip of last model varies in pre and postpeak (0.3 < slip < 1 mm). The top subplot shows that the base numerical model has the lowest yarn force distribution due to low bond stress while the modified model with springs has higher force due to additional restraint forces from the springs. The modified model with slack yields the highest force; however, this force is still less than the other two models at peak pullout load when slip reaches 0.5 mm [Fig. 3.23(a)].

3.3.3.2. Effect of Bond Strength, Degree of Slack and Transverse Yarn Strength

The second set of parametric study is shown in Fig. 3.24, which demonstrates the effect of bond strength, degree of slack in longitudinal yarn and junction strength to the pullout load-slip response. Numerical models presented in Table 3.6 that input material models are varied in three levels are used for this study. It can be seen from Fig. 3.24(a) that the increase in bond strength from the base level by 25% and 50% results in proportional increase of peak load (35.5/35.5 = 1.00, 44.5/35.5 = 1.25 and 53.1/35.5 = 1.50). It can be noticed that the shapes of the pullout slip responses are resemble to the shape of input bond slip models. Fig. 3.24(b) reveals that the effect of slack in longitudinal yarn makes
apparent stiffness and ultimate capacity lower than those of the base numerical model. Fig. 3.24(c) confirms the study in previous section that the presence of cross yarn junctions increases the pullout resistant. As the strength of springs increased by 1, 2 and 3 times the base level, the ultimate load increases in a decreasing rate (35.5/35.5 = 1.00, 43.9/35.5 = 1.24 and 52.1/35.5=1.47 and 59.6/35.5 = 1.68). This is due to the fact that bond and spring contribute to the pullout response independently according to the slip level and material models specified.

3.3.4. Comparison with Other Fiber Pullout Models

In the verification process, the finite difference fabric pullout model was compared to an analytical fiber pullout model using N-piecewise linear bond law proposed by Banholzer (2005a). An experimental result of pulling a steel rebar out of a concrete cylinder was used. The description of the bond model and other parameters for simulation are shown in Fig. 3.25(a); the embedded length, \(L = 40\) mm; rebar diameter, \(\phi = 16\) mm; Young’s modulus of steel, \(E_s = 210\) GPa; Young’s modulus of concrete matrix, \(E_m = 35\) GPa, and its cross section area \(A_m = 10000\) mm\(^2\). In the case of steel rod, slack and spring were not used in the finite difference model. It should be noted that the analytical model considers the concrete strain while the proposed numerical model ignores this parameter in the simulation.

Fig. 3.25(b) shows that the predicted response of the analytical model agrees very well with the experimental result as the inputted bond parameters were obtained from material calibration of the experiment. The finite difference model using the same bond parameters agrees well with the analytical model and experimental result up to the peak
but slightly less stiff in the postpeak response. This is due to the difference in assumptions between the two methods. In derivation of analytical solution, an extruding part is still have bond stress as described by the bond slip relation while the finite difference model assumes this part has zero bond stress. Less resistant bond stress yields softer response, which becomes more evident when the extruding part becomes longer. Note that in the present case, the axial stiffness ratio between concrete matrix and steel rebar \((A_mE_m/A_sE_s = 350/42.2)\) is relatively high at 8.3, therefore the simplified assumption used in the proposed model that neglects the matrix strain still give good prediction of the pullout response.

### 3.3.5. Case Study of AR-Glass Fabric Reinforced Cement

Four sets of AR-glass samples G105, GP105, GV105 and FG105 (Table 3.3) were previously analyzed using a single yarn pullout model in section 3.2 to obtain equivalent bond slip parameters, which are shown in Fig. 3.26(a) and Table 3.7. The simulation responses of these four fabrics in Fig. 3.13(a) are repotted with their experimental results and presented in Fig. 3.26(b). All specimens had the same embedded length of 12.7 mm. Fig. 3.27(a) shows the plain fabric in loading direction (longitudinal) and transverse direction. The fabric contains five pairs of yarn per inch in each direction. Each yarn consists of 400 glass filaments, 13.5 microns in diameter each and they are bundled to form an equivalent diameter of 0.27 mm in longitudinal direction. The experimentally obtained averaged modulus of the fabric in longitudinal direction was 58,605 MPa (Table 3.3), which was less than theoretical modulus of individual glass filaments (78,600 MPa in Table 3.2) making up the yarn. Fig. 3.27(b) presents an SEM picture of
the glass fabric embedded in cement paste matrix, in which the longitudinal yarn is bonded to the flat transverse yarn. With the new tool developed in this section, the four glass samples will be reanalyzed using the finite difference fabric pullout model, which allows for better simulation of pullout response and characterization of bond properties. The stiffness efficiency factors $\eta$ identified from previous study for G105, GP105, GV105 and FG105 were of 5.0, 5.7, 9.5 and 4.0 %, respectively, and these values were again used to reduce the axial stiffness of longitudinal yarn ($\eta A_y E_y$) in the finite difference model.

3.3.5.1. Numerical Investigation by Fabric Pullout Model

Figs. 3.28(a and b) show a schematic drawing of an AR-glass fabric reinforcement cement and its representative numerical model. The embedded length of 12.7 mm was discretized into 1017 nodes. Two lines of transverse yarns, spaced 5.08 mm apart, were modeled as two springs; each has equivalent strength of 8 cross yarn junctions. In simulations, it assumed that the springs were placed symmetrically in the center region. Eight longitudinal yarns pulled out were represented by the equivalent area and perimeter of 8 times the individual yarn $A_y$.

To find material parameters for the spring model, a plain AR glass fabric was tested by pulling a longitudinal yarn until the junction failure. It was found that the junction failed in brittle manner with an average strength of 9.0 N per junction (or the total of 72 N for 8 junctions). Since the fixity at the junction surrounded by cement paste is relatively stiff [Fig. 3.27(b)] compared to the load capacity of the junctions (9 N), it assumes that spring force response begins immediately without any delay and drops to
10% of the ultimate capacity at an arbitrary slip in the postpeak region. The force continues to decrease and vanish when the junction slips by more than a width of a transverse yarn (1.78 mm). Finally, the inverse analysis was performed to find the best set of material parameters for bond and spring models that made the pullout simulations fit to the experimental results.

3.3.6. Results and Discussions

Figs. 3.29(a and b) show the obtained bond and spring models from the inverse analysis. The bond parameters are also given in Table 3.8. Figs. 3.30 (a - d) show the simulations of pullout responses of the four AR-glass samples due to bond along the yarn only and due to both bond and springs at the junctions. It can be seen from Figs. 3.30(a - c) that the responses consist of two peaks in which the first one is higher than the second one. By characterizing with the finite difference fabric pullout model, it can be identified that the first peak of the response is contributed by bond and spring while the second peak is contributed solely by the bond model.

The bond parameters previously analyzed in Table 3.7 cannot be directly compared to the new analysis results presented in Table 3.8 since the first set is equivalent bond parameters including the junction effects while the second set is true bond parameters exist in longitudinal direction of the yarn. Nevertheless, both characterizations yield the same conclusion that the bond of mixtures containing fly ash (FG105), or those subjected to vacuum processing (GV105) are higher than the cast (G105) and pultrusion techniques (GP105). Compare the simulations of pullout responses between Fig. 3.30 and Fig.
3.26(b), ones can see that the proposed fabric pullout model can capture the responses better than the yarn pullout model using equivalent bond parameters.

3.4. Chapter Conclusions

An analytical fabric pullout model based on Naaman et al. (1991a,b) was used to investigate the bond mechanism expressed as a shear strength diagram. The model has demonstrated the ability to fit various representative curves (averaged responses) of different fabrics, matrices, embedded length, and processing methods. The maximum shear strength $\tau_{\text{max}}$ and frictional shear strength $\tau_{\text{frc}}$ are correlated well with the nominal shear strength $\tau_{\text{nom}}$. In order to simulate the experiment pullout responses, a parameter representing the efficiency of the yarn stiffness $\eta$ was introduced. It was observed that the pullout responses are highly dependent on the fabric type, mixture design, and processing methods. The glass fabric exhibits the best bond with the cement matrix, mainly when a high content of fly ash is used. The pultrusion technique increases bond strength where noncoated multifilament was used to produce the fabric (PP and PVA) while the vacuum technique increases in AR-glass bond but is ineffective with PP. It was concluded that the processing methods can highly affect the bond between fabrics and cement matrix, depending on the nature and geometry of the fabric and its yarns.

A fabric pullout model based on a nonlinear finite difference method was also developed to be used in material characterization of fabric-matrix interface and the resistance due to cross yarn junctions. A universal free-form material model was used to describe arbitrary nonlinear characteristics of interface, longitudinal yarns and the mechanical anchorage offered by transverse yarns. The parametric studies show that the
effect of increasing bond strength proportionately increases the pullout strength. Slack in longitudinal yarn causes an apparently weaker and more flexible pullout response. With a presence of a bonded fill yarn to serve as mechanically anchor to the longitudinal yarn, a substantially increases in pullout strength was observed. In the verification process, the finite difference fabric pullout model using the same bond parameters as those used in N-piecewise linear bond slip relation predicted the responses close to the experimental result of pulling a steel rod out of concrete matrix. The developed model was then used to analyze the test results of AR-glass fabric reinforced cement to quantitatively identify the contribution between bond and cross yarn junctions. It is clearly shown that the first phase of pullout resistance was contributed by bond and springs while the second phase which was after the sequential failure of the springs was due to bond only.

Notation

*The following symbols are used in this paper:*

- $A =$ cross sectional area;
- $B =$ bond resistance force;
- $[C] =$ coefficient matrix in system of equations;
- $D_{measured}(L) =$ measured sliding slip from experiment in dynamic stage III;
- $d =$ debonded length;
- $dx =$ finite length of yarn in matrix;
- $E =$ Young modulus;
- $F =$ force in yarn;
- $G =$ spring force;
\( g \) = secant modulus of spring force-slip model;
\( h \) = finite length of yarn between two adjacent nodes;
\( K \) = slope of pullout curve;
\( k \) = secant modulus of bond-slip model;
\( L \) = embedded length;
\( M \) = force in matrix;
\( N \) = number of yarns pulled or number of replicate samples;
\( n \) = number of nodes in finite element model;
\( P \) = pullout load;
\( \{P\} \) = nodal pullout load vector;
\( P_{1b,\text{max}} \) = maximum bonded load at the end of Stage I;
\( P_{2b,\text{max}} \) = maximum bonded load at the joining bonded–debonded zones in Stage II;
\( P_{\text{max}} \) = maximum pullout load;
\( Q \) = \( \frac{1}{E_y} + \frac{1}{E_m} \);
\( SF_{\text{max opt}} \) = optional spring model that can be attached to the base numerical model;
\( s \) = slip at matrix yarn interface;
\( \{S\} \) = nodal slip vector;
\( s(L) \) = end slip of the yarn;
\( s_{\text{total}}(L) \) = total (accumulated) end slip of the yarn;
\( s' \) = derivative of slip with respect to longitudinal axis \( x \);
\( s^* \) = imposed slip at the end of yarn;
\[ \beta = (\psi \kappa Q)^{1/2}; \]

- \( \Delta_d \) = rigid body sliding of yarn;
- \( \delta \) = elongation;
- \( \varepsilon \) = strain;
- \( \varepsilon_{\text{slack, opt}} \) = optional slack parameter at beginning of the yarn stress strain model;
- \( \phi \) = diameter;
- \( \eta \) = axial stiffness efficiency factor for yarn;
- \( \kappa \) = slope of shear strength diagram;
- \( \mu \) = mean load slip response;
- \( \sigma \) = standard deviation of load-slip response;
- \( \tau \) = shear stress at yarn matrix interface;
- \( \tau_{\text{dyn}} \) = dynamic shear strength;
- \( \tau_{\text{frc}} \) = frictional shear strength;
- \( \tau_{\text{max}} \) = maximum shear strength;
- \( \tau_{\text{max, base}} \) = base bond-slip model used in numerical simulation;
- \( \tau_{\text{nom}} \) = nominal shear strength;
- \( \xi_b, \xi_{E_y}, \xi_g \) = weighting factors for calculating next secant modulus for bond-slip;
  - yarn stress strain and spring force-slip models, respectively;
- \( \psi \) = equivalent circumference of yarn.

**Subscripts**

- \( m, s, y \) = matrix, steel, yarn;
1 = fully bonded Stage I;
2 = debonding Stage II;
2, last = at the end of debonding Stage II;
3, 1st = at initial dynamic Stage III;
3, nth = sub sequential dynamic Stage III;
i + 1/2 = at half spacing to the right of node i;
i - 1/2 = at half spacing to the left of node i; and
ref = reference secant modulus used in updating material stiffness.
### Table 3.1. Mix Designs of the Different Mixtures

<table>
<thead>
<tr>
<th>Ingredients</th>
<th>Control Mix</th>
<th>Fly Ash Mix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>829 g</td>
<td>829 g</td>
</tr>
<tr>
<td>Cement</td>
<td>2040 g</td>
<td>1224 g</td>
</tr>
<tr>
<td>Silica Fume</td>
<td>169 g</td>
<td>169 g</td>
</tr>
<tr>
<td>Fly Ash</td>
<td>-</td>
<td>816 g</td>
</tr>
<tr>
<td>Superplasticizer</td>
<td>5.25 ml</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 3.2. Properties of the Fibers Made up the Fabrics

<table>
<thead>
<tr>
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<th>Yarn Nature</th>
<th>Strength of Elasticity</th>
<th>Modulus of Elasticity</th>
<th>Strain at Peak</th>
<th>Filament Size</th>
<th>Number of Filaments in a Bundle</th>
<th>Approx. Bundle Diameter</th>
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<td>1276-2448</td>
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Table 3.3. Material Parameters Used in Simulations

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<th>and Processing</th>
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<th>$L$ (mm)</th>
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<td>FGP</td>
<td>FA, pultrusion</td>
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<td>PVA</td>
<td>FA, pultrusion</td>
<td>7.6</td>
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<tr>
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<td>PVA</td>
<td>FA, pultrusion</td>
<td>12.7</td>
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</tbody>
</table>

*Note that*

FA = fly ash and the cement block has a dimension of 8.1x25.4 with an assumed Young modulus of 10342 MPa and Poisson ratio of 0.2 for all test series.
### Table 3.4. Key Parameters Obtained from Experiments and Simulations

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<tr>
<th>Series</th>
<th>Set of data</th>
<th>$K$</th>
<th>$P_{max}$</th>
<th>$\tau_{nom}$</th>
<th>$\kappa$</th>
<th>$\tau_{max}$</th>
<th>$\tau_{frc}$</th>
<th>$\tau_{dyn}$</th>
<th>$\eta$</th>
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<tr>
<td>G105</td>
<td>Exp.</td>
<td>N/mm</td>
<td>N</td>
<td>N-mm</td>
<td>MPa</td>
<td>N/mm$^2$</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td>210(70)</td>
<td>186(18)</td>
<td>0.74(0.15)</td>
<td>67(25)</td>
<td>2.16(0.21)</td>
<td>4.24</td>
<td>3.41</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>Rep.</td>
<td>239</td>
<td>170</td>
<td>0.72</td>
<td>63</td>
<td>1.97</td>
<td>4.24</td>
<td>3.41</td>
<td>1.24</td>
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<tr>
<td>GP105</td>
<td>Exp.</td>
<td>281(73)</td>
<td>168(25)</td>
<td>0.69(0.26)</td>
<td>64(25)</td>
<td>1.94(0.29)</td>
<td>2.96</td>
<td>2.96</td>
<td>1.24</td>
</tr>
<tr>
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<td>154</td>
<td>0.67</td>
<td>62</td>
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<tr>
<td>GV105</td>
<td>Exp.</td>
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<td>250(36)</td>
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<td>85(14)</td>
<td>2.91(0.41)</td>
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<td>4.24</td>
<td>2.96</td>
</tr>
<tr>
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<td>Rep.</td>
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<td>FG105</td>
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<td>251(80)</td>
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<td>227(63)</td>
<td>1.45(0.43)</td>
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<td>2.63(0.73)</td>
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<td>4.24</td>
<td>2.96</td>
</tr>
<tr>
<td></td>
<td>Rep.</td>
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<td>215</td>
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<td>147</td>
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<td>7.13</td>
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<td>1.79</td>
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<tr>
<td>PP103</td>
<td>Exp.</td>
<td>234(137)</td>
<td>130(39)</td>
<td>1.34(0.75)</td>
<td>112(61)</td>
<td>1.70(0.51)</td>
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</tr>
<tr>
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<td>4.24</td>
<td>2.96</td>
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<td></td>
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<td>178</td>
<td>1.72</td>
<td>221</td>
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<td>4.24</td>
<td>2.76</td>
<td>1.05</td>
</tr>
<tr>
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<td>227(60)</td>
<td>2.00(0.32)</td>
<td>294(118)</td>
<td>2.96(0.78)</td>
<td>4.24</td>
<td>4.76</td>
<td>2.14</td>
</tr>
<tr>
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<td>Rep.</td>
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<td>219</td>
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<td>239</td>
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<td>4.76</td>
<td>2.14</td>
</tr>
<tr>
<td>PPP105</td>
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<td>308(47)</td>
<td>3.10(0.17)</td>
<td>587(106)</td>
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<td>135(12)</td>
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<td>1.06(0.10)</td>
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<tr>
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<td>17(3)</td>
<td>76(6)</td>
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<tr>
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<td>144(35)</td>
<td>1.32(0.16)</td>
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<td>0.94(0.23)</td>
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<td>4.24</td>
<td>2.96</td>
</tr>
<tr>
<td></td>
<td>Rep.</td>
<td>116</td>
<td>140</td>
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</tr>
<tr>
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<td>Rep.</td>
<td>215</td>
<td>219</td>
<td>1.22</td>
<td>156</td>
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<td>0.34</td>
<td>1.62</td>
<td>1.31</td>
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<td>Exp.</td>
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<td>1.70</td>
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**Note that**

Exp. and Rep. in column “Set of data” refer to the results from experiment of 4 samples (except Exp*. 3 samples) and the results from a single representative curve.
**Table 3.5. Upper and Lower Bound Shear Strength Parameters for Series PP105**

<table>
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<tr>
<th>Response</th>
<th>( \kappa )</th>
<th>( \tau_{\text{max}} )</th>
<th>( \tau_{\text{frc}} )</th>
<th>( \tau_{\text{dyn}} )</th>
<th>( \eta )</th>
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</thead>
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<tr>
<td>( \mu - \sigma )</td>
<td>3.90</td>
<td>2.62</td>
<td>0.91</td>
<td>0.83</td>
<td>15</td>
</tr>
<tr>
<td>( \mu )</td>
<td>4.24</td>
<td>2.76</td>
<td>1.05</td>
<td>0.90</td>
<td>17</td>
</tr>
<tr>
<td>( \mu + \sigma )</td>
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<td>3.34</td>
<td>1.17</td>
<td>1.00</td>
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**Table 3.6. Numerical Models for Parametric Studies and the Simulation Results**

<table>
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<th>Model Name</th>
<th>Input Material Models</th>
<th>Predicted Responses</th>
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<td>Bond Model</td>
<td>Yarn Model</td>
</tr>
<tr>
<td></td>
<td>( \tau_{\text{max}} ) (MPa)</td>
<td>( \varepsilon_{\text{slack}} ) (mm/mm)</td>
</tr>
<tr>
<td>Base Model</td>
<td>2.0</td>
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</tr>
<tr>
<td>1.25 ( \times ) ( \tau_{\text{max}}_{\text{base}} )</td>
<td>2.5</td>
<td>-</td>
</tr>
<tr>
<td>1.50 ( \times ) ( \tau_{\text{max}}_{\text{base}} )</td>
<td>3.0</td>
<td>-</td>
</tr>
<tr>
<td>+ 1.0 ( \times ) ( \varepsilon_{\text{slack opt}} )</td>
<td>2.0</td>
<td>+ 0.050</td>
</tr>
<tr>
<td>+ 1.5 ( \times ) ( \varepsilon_{\text{slack opt}} )</td>
<td>2.0</td>
<td>+ 0.075</td>
</tr>
<tr>
<td>+ 2.0 ( \times ) ( \varepsilon_{\text{slack opt}} )</td>
<td>2.0</td>
<td>+ 0.100</td>
</tr>
<tr>
<td>+ 1 ( \times ) ( SF_{\text{max opt}} )</td>
<td>2.0</td>
<td>-</td>
</tr>
<tr>
<td>+ 2 ( \times ) ( SF_{\text{max opt}} )</td>
<td>2.0</td>
<td>-</td>
</tr>
<tr>
<td>+ 3 ( \times ) ( SF_{\text{max opt}} )</td>
<td>2.0</td>
<td>-</td>
</tr>
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### Table 3.7. Equivalent Bond Slip Parameters for a Smooth Yarn Pullout Model

<table>
<thead>
<tr>
<th></th>
<th>Glass-Cast (G105)</th>
<th>Glass-Vacuum (GV105)</th>
<th>Glass-Pultrusion (GP105)</th>
<th>Glass-Flyash (FG105)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s (mm)</td>
<td>τ (MPa)</td>
<td>s (mm)</td>
<td>τ (MPa)</td>
<td>s (mm)</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
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<td>3.96</td>
<td>0.46</td>
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<tr>
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<td>2.07</td>
<td>0.46</td>
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<td>1.24</td>
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<td>2.07</td>
<td>0.71</td>
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<tr>
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<td>0.65</td>
<td>2.62</td>
<td>0.71</td>
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<td>1.59</td>
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### Table 3.8. Bond-Slip Parameters for a Finite Difference Fabric Pullout Model

<table>
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<th>Glass-Vacuum (GV105)</th>
<th>Glass-Pultrusion (GP105)</th>
<th>Glass-Flyash (FG105)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s (mm)</td>
<td>τ (MPa)</td>
<td>s (mm)</td>
<td>τ (MPa)</td>
<td>s (mm)</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.15</td>
<td>0.95</td>
<td>0.20</td>
<td>1.60</td>
<td>0.13</td>
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<td>1.86</td>
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<tr>
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<td>12.7</td>
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</tbody>
</table>


Fig. 3.1. Scanning electron microscope (SEM) pictures of different fabric reinforced cement composites; (a) nonlinear geometry of longitudinal yarns and cross yarn junctions; (b) tortuous crack propagation; (c) matrix densification
Fig. 3.2. Effect of manufacturing techniques to the tensile stress strain response of fabric reinforced cement made of 2 layers of Saint Gobain Technical Fabrics
Fig. 3.3. Pullout load slip response of Polypropylene (PP) embedded as a yarn and as a fabric
Fig. 3.4. Fabric pullout setup

Fig. 3.5. Fabric pullout-slip response at difference stage of debonding
Fig. 3.6. Schematic drawing of yarn pullout test: (a) side view of the upper half model; (b) specimen cross section

Fig. 3.7. Shear strength diagram
Fig. 3.8. Shear stress and force distribution along the yarn: (a) Stage I (elastic response); (b) Stage II (nonlinear response); (c) Stage III (dynamic response); (c.1) first dynamic responses (no sliding, $\Delta d = 0$); (c.2) subsequent dynamic responses (when sliding occurs, $\Delta d > 0$)
Fig. 3.9. Schematic description of the different fabrics: (a) bonded; (b) knitted; (c) woven; (d) four fabric types used (number of yarns inside/outside cement block)
Fig. 3.10. Casting of fabric pullout specimens
Fig. 3.11. Fabric pullout specimens after cutting
Fig. 3.12. Experimental pullout responses prior and after adjustment for free fabric length and initial deformation including the representative curve for each control test series: (a) G105; (b) PP105; (c) PE105; (d) PVA103
Fig. 3.13. Representative pullout curves (averaged responses of 4 samples) of all 15 test series used in simulations of the different fabrics: (a) AR-Glass (G) series; (b) Polypropylene (PP) series; (c) Polyethylene (PE) series; (d) Polyvinyl Alcohol (PVA) series
Fig. 3.14. Simulation of representative pullout curve for each control test series: (G105, PP105, PE105 and PVA103)
Fig. 3.15. Simulation of pullout responses for control series PP105 (average, lower and upper bound). The upper and lower bound responses are the averaged response + 1 standard deviation.
Fig. 3.16. Shear strength parameters: (a) averaged nominal shear strength from experiments vs. maximum and frictional shear strength from best fit of simulations; (b) averaged nominal shear strength from experiments with standard deviation.
Fig. 3.17. Correlation of shear strength parameters: nominal shear strength ($\tau_{nom}$), maximum shear strength ($\tau_{max}$) and frictional shear strength ($\tau_{frc}$).
Fig. 3.18. Schematic drawing of fabric pullout mechanism; (a) interface bond-slip model; (b) longitudinal yarn stress-strain model; (c) spring force-slip model simulating anchorage strength at cross yarn junction; (d) fabric pullout specimen
Fig. 3.19. Finite difference fabric pullout model; (a) discretized fabric pullout model under displacement control; (b)-(d) free body diagram of three typical nodes “A” - “C”
Fig. 3.20. An algorithm for updating material stiffness and checking for convergence of local force deformation at each node for each imposed slip increment.
Fig. 3.21. Numerical fabric pullout specimen for parametric studies
(a) \( \tau_{\text{max\_base}} = 2 \text{ MPa} \)

(b) \( \varepsilon_{\text{slack\_opt}} = 0.05 \)
Fig. 3.22. Material constitutive laws; (a) bond stress slip model; (b) longitudinal yarn stress strain model; (c) spring force slip model
Slip (mm) vs. Embedded Length 12 mm

- Bond Stress (MPa)
- Yarn Force (N)

Legend:
- $+1 \times SF_{\text{max_opt}}$
- $+1 \times \varepsilon_{\text{slack_opt}}$
- Base Model

(a)
Fig. 3.23. Compare distribution of slip, bond stress and yarn force along the embedded length; (a) and (b) imposed slip at the right end reaches 0.5 mm and 1.0 mm
Graph (a) shows the load (N) as a function of slip (mm) for different normalized values of the sliding friction force $\tau_{\text{max base}}$:
- $1.50 \times \tau_{\text{max base}}$
- $1.25 \times \tau_{\text{max base}}$
- Base Model

Graph (b) illustrates the load (N) as a function of slip (mm) for different normalized values of $\varepsilon_{\text{slack opt}}$:
- $+2.0 \times \varepsilon_{\text{slack opt}}$
- $+1.5 \times \varepsilon_{\text{slack opt}}$
- $+1.0 \times \varepsilon_{\text{slack opt}}$
- Base Model
Fig. 3.24. Parametric studies of material models to pullout load-slip responses; (a) the effect of bond strength; (b) the effect of slack; (c) the effect of spring strength
Fig. 3.25. Comparison of pullout responses of steel rebar in normal strength concrete between analytical and numerical models; (a) the bond stress slip relation for both models; (b) the predicted pullout responses
Fig. 3.26. Simulation of fabric pullout using equivalent single yarn pullout model; (a) best fit bond slip parameters; (b) simulations of the experimental results
Fig. 3.27. AR-glass fabric used in pullout test; (a) plain fabric; (b) fabric inside cement matrix
Fig. 3.28. Modeling of AR-Glass fabric reinforced cement; (a) pullout specimen; (b) representative numerical model
Fig. 3.29. Calibrated material parameters for AR-glass fabric reinforced cement; (a) best fit bond slip model; (b) best fit spring model
(a) Glass Cast (G105)

(b) Glass Pultrusion (GP105)
Fig. 3.30. Simulation of pullout response of AR-glass fabric; (a) cast sample, (b) pultrusion sample; (b) vacuum process sample; (d) mix with flyash sample.
CHAPTER 4

TENSILE MODELING OF FIBER REINFORCED CEMENT COMPOSITES

4.1. Introduction

Tensile behaviors of fiber reinforced cement based composites are primarily governed by three to four main factors: bond-slip characteristics at interface, cracking strength of matrix, stress strain behavior of reinforcing materials and may includes the resistant provided by transverse reinforcement. Under service loads most cement based composites are subjected to minor to moderate cracks. Therefore, their tensile characteristics in uncracked and postcrack stages are of particular interest for general uses of the composites.

At low tension force concrete matrix and reinforcing materials together act as a composite material in carrying the applied load. At higher load level exceeding cracking limit of the matrix, the composite cracks and bond mechanism play an important role in transferring the load from reinforcing materials to the uncracked portion of the matrix. The ability of the uncracked segments to carry tensile force is known as “tension stiffening”; the higher the tension stiffening the stiffer the postcrack response of the cracked composite. Another important aspect of composite structures under service load is the crack evolution, which includes crack spacing and crack widening. Fewer cracks but larger opening makes structural appearances unpleasant and vulnerable to ingress of corrosive agents. Toughening ability that scatters a single large crack to multiple cracks invisible to naked eye is preferable. Several analytical models have been proposed to explain the load-elongation behavior and crack formations in concrete composites.
Most analytical models are applicable for a specific pair of fiber and matrix according to their specific material models used in derivations. Different assumptions used among those analytical models also cause inconsistency of material parameters obtained from calibration processes. These make it difficult to study and compare toughening mechanisms of various fiber reinforced composites. A more powerful finite element method can be used to model interface and crack related problems in composite structures; however, the method requires extensive knowledge in numerical analysis and cracking mechanisms in brittle materials (Pamin and de Borst 1998; Soltani et al. 2003, 2004; Holler et al. 2004; Haussler-Combe and Hartig 2007). The computational cost of this approach is enormous and not practical to be used as material characterizations of cement based composites.

A relatively simpler finite difference approaches provides an alternative in modeling of tension stiffening. The finite difference fabric pullout model developed in the previous chapter is extended in this chapter in order to model the force transfer mechanisms from fabric to matrix and the crack evolution in specimens. Even though the model is developed for fabric reinforced cement composites, the algorithm is general and can be used to simulate tensile behavior of several cementitious composites. However, the applicability of this model is limited to the assumption that the axial stiffness of the matrix must dominate the axial stiffness of the reinforcing materials. This assumption is normally satisfied in lightly fiber or fabric reinforced composites.
4.2. Tension Stiffening in Fabric Reinforced Cement Composites

Typical tension specimens of fabric reinforced cement composites made of four materials [AR-glass (G), Polypropylene (PP), Polyvinyl Alcohol (PVA) and polyethylene (PE)] are presented in Fig. 4.1. The composite consists of cement paste matrix and eight layers of fabrics. The ends of the specimen are normally glued to aluminum plates to prevent cracking in the grip zones when it is pulled by the testing machine. The clear length between two aluminum plates is used to measure material responses such as elongation, crack spacing and crack width, which are then analyzed to obtain engineering material properties.

The proposed tension stiffening model divides the tensile behavior into two stages: uncracked and postcrack as shown in Fig. 4.2. When the composite is loaded below its cracking limit, fiber and matrix are assumed to be fully bonded without slip. Thus, in this range, rule of mixture and strain compatibility are applied. Strains in yarn, matrix and composite are all equal and the strength contributions of the two materials depend on their relative axial stiffness. Once the specimen is loaded beyond cracking limit, crack evolution in specimen takes place; number of cracks increases and crack spacing decreases. Each cracked segment can be visualized as a pullout problem subjected to the same applied load at crack locations. The process continues until the stress in cracked segments can not develop to the matrix cracking strength. Therefore, number of cracks in specimen is stabilized. As load continues to increase the slip at crack locations continues to increase, leading to crack widening. The primary strength contribution at this stage primarily comes from the reinforcing fabrics. When the tension load approaches the
ultimate capacity, the weaker yarns start to break, causing force redistribution to the remaining yarns to carry higher load. Since fabrics are relatively brittle, their failure mechanics causes brittle failure of the composite after the ultimate load.

According to homogenization concept that materials are reinforced uniformly throughout the cross section, this class of composite is normally characterized by nominal tensile stress and strain. The stress is obtained by dividing the applied load with the gross section area. The strain is obtained by summing all crack widths and adding elastic elongation of cracked segments, then dividing the total elongation with the gage length used in the measurement.

4.2.1. Material Constitutive Behaviors and Cracking Criterion

Fig. 4.3 presents a schematic drawing of a cracked fabric reinforced concrete, which is controlled by four mechanical aspects: bond-slip characteristics at interface; tensile stress strain behaviour of the longitudinal yarn, mechanical anchorage provided by cross yarn junctions and cracking criterion defined by matrix strength.

Bond between yarn and cementitious matrix can be described by bond stress $\tau$ as a function of slip $s$, which can be approximated by a free form model [Fig. 4.3(a)] consisting of several linear segments to define the pre and postpeak behavior of the bond characteristics. At each load step, a secant modulus $k$ is used to enforce the local bond stress and slip at each node in the finite difference model to follow the specified bond slip relation.

The second aspect is slack that is commonly found in the composites prepared by hand lay-up method. Without pre-tensioning during casting the samples, the fabric is crimped
and left slack in the harden matrix. Shrinkage in high cement content matrix that shortens the original length of the fabric also adds more slack into the system. This aspect can be incorporated into a longitudinal yarn stress strain model [Fig. 4.33(b)] by adding an initial delay of load deformation response before the beginning of the elastic part. After the end of linear elastic portion, the response becomes highly nonlinear until it breaks at the ultimate strength. Similarly, the secant modulus $E_y$ is used to enforce the longitudinal yarn stress strain model used in the finite difference model.

The third aspect of the modeling is the additional pullout resistant provided by the cross yarn junction at the intersection of longitudinal and transverse yarns. Several fabric structures such as woven, bonded and knitted may have different load transfer mechanisms from longitudinal yarn to the junction and the junction to the surrounding matrix. This behavior can be simplified to an equivalent nonlinear spring that express the resistant force as a function of slip. Fig. 4.3(c) shows a possible shape of a spring force – slip model that the response may begin with some delay due to slack in fabric and continue with linear elastic up to the peak before it fails in a relative brittle manner. Similar to the previous two models, the secant modulus $g$ is used to control local spring force and slip at nodes.

The last aspect is the cracking criterion defined by a single value of matrix strength $\sigma_{mcr}$ as shown in Fig. 4.3(d). Uniform or non-uniform strength distribution along the length of specimen can be generated by any probabilistic density function. Deterministic and stochastic crack patterns can be simulated by manipulating matrix strength distribution. To achieve the deterministic crack pattern [Fig. 4.4(a)] that the first crack
appears at the center, breaking the specimen into two half, then at the edge of end grips, quarter, one eighth, so on and so for, a uniform strength must be specified everywhere, except slightly lower (less than 0.01%) at the desired sequentially crack locations (center, grip edges, quarter, one-eighth, etc). Note that the decrease in strength is very small; thus, it can be said that the specimen has a uniform strength everywhere. At the grips, matrix is strengthened by metal plates and crack is prohibited in these regions. Therefore, the crack detection algorithm ignores the possibility of stress exceeding cracking strength at the grips. On the contrary, the stochastic crack pattern can be achieved by generating random strength distribution as shown in Fig. 4.4(b) such that cracks appear randomly at locations where stress exceeds the local strength. If the minimum matrix strength is slightly less than the mean strength (less than 0.01%), the perturbation in strength distribution only causes random crack pattern but the specimen can be said to have a uniform strength. Likewise, no crack is allowed in the end zones for random crack pattern.

With similarity of the first three material models that use free form functions to describe the material characteristics and employ secant modulus in controlling force deformation to follow material laws, the first three material models can be implemented in a finite difference model as a unified constitutive law while the last model is used as a cracking criterion.

4.2.2. Finite Difference Tension Stiffening Model

Cracked tension specimen can be idealized to 1D series of pullout segments. Since concrete matrix is brittle, the matrix stress at a crack location is zero and the load is solely
carried by the longitudinal yarn. Thus, the tension problem can be solved as an individual pullout segment subjected to the same load applied at crack stations. Once all segments are solved, the solutions are then combined to represent the response of the whole tension specimen. Two basic concepts of equilibrium and material constitutive behaviors are used in the formulation of the pullout problem. The equilibrium equations can be derived from free body diagram at a node while the material constitutive behaviors are enforced by secant modulus in the iterative material updating process.

4.2.2.1. Finite Difference Equilibrium Equations

In derivation of equilibrium equations, slip which is used as an unknown variable in the system of equations is defined as the relative difference between the elongation of the longitudinal yarn and matrix.

\[ s = \int (\varepsilon_y - \varepsilon_m) \, dx \] (4.1)

where \( \varepsilon_y \) and \( \varepsilon_m \) are yarn and matrix strain, respectively. For typical low fiber volume fraction, the axial stiffness of the yarn \( A_y E_y \) is considerably lower than the axial stiffness of the matrix \( A_m E_m \) and the effect of matrix strain to slip can be ignored. Thus, the slip \( s \) and yarn strain \( \varepsilon_y \) are simplified to:

\[ s = \int \varepsilon_y \, dx \quad \text{and} \quad \varepsilon_y = s' = \frac{ds}{dx} \] (4.2)

where \( dx \) is a finite length in a numerical model. Fig. 4.5(a) presents a finite difference model of a cracked specimen. The total embedded length \( L \) is discretized into \( N \) nodes with equal spacing of \( h \). Optional springs can also be attached to the nodes at cross yarn junctions to provide additional resistant to pullout force. Once matrix cracks, the whole
length breaks into smaller segments $L_s^{(1)}, L_s^{(2)}, \ldots$ and each contains $n$ number of local nodes. It should be noted that an additional node is inserted at the crack location such that each cracked segment has its own end nodes and the problem can be solved independently.

Fig. 4.5(b) shows slip distribution in cracked segments, which assume positive for the slip moving to the right and negative to the left. Since the secant modulus of the material models [Fig. 4.3(a)-(c)] are defined as positive quantities, the directions of bond stress and spring force must follow the same sign convention as slip. On the other hand, tension force in longitudinal yarn is always defined as a positive value. Two boundary conditions at the ends of each segment are required to solve a pullout problem. At the left segment of Fig. 4.5(a and b), the force in longitudinal yarn at the local node 1 must be zero, which implies the yarn strain or derivative of slip with respect to $x$ must be zero. At the right end of the left segment, the force must be equal to the applied load $P$. In the middle segment, force in the longitudinal yarn at each end (local node 1 and $n$) must be equal to the same applied load $P$. Finally, the boundary conditions of right segment are the same as the left segment but in the reverse order. Considering force equilibrium in the domain and boundary conditions at the ends of each segment, six representative nodes labeled as “A”, “B”, “C”, “D”, “E” and “F” in Fig. 4.5(a) are needed in formulation of finite difference equations and their free body diagrams are represented in Fig. 4.5(c). The equations for typical nodes “A”, “B” and “C” had been derived in the previous chapter, and they are repeated here for convenience of reference. The full derivations of the remaining three nodes “D”, “E” and “F” will be presented in this chapter.
4.2.2.1. Boundary at the Left End

The equilibrium equation for the representative node “A” (local node 1 of the left segment) is

\[- \left( A_y \left( \frac{E_{y,1} + E_{y,2}}{2} + \frac{1}{2} \psi k_l h^2 \right) \right) s_1 + A_y \left( \frac{E_{y,1} + E_{y,1}}{2} \right) s_2 = 0 \quad (4.3)\]

4.2.2.1.2. Interior Nodes

Two equilibrium equations for the representative node “B” and “C” (interior nodes with and without spring) are

\[- A_y \left( \frac{E_{y,i-1} + E_{y,i}}{2} \right) s_{i-1} - A_y \left( \frac{E_{y,i-1} + E_{y,i+1}}{2} \right) + \psi k_i h^2 + g_i h_i \] 
\[+ A_y \left( \frac{E_{y,i} + E_{y,i+1}}{2} \right) s_{i+1} = 0 \quad (4.4)\]

\[- A_y \left( \frac{E_{y,i-1} + E_{y,i}}{2} \right) s_{i-1} - A_y \left( \frac{E_{y,i-1} + E_{y,i+1}}{2} \right) + \psi k_i h^2 \] 
\[+ A_y \left( \frac{E_{y,i} + E_{y,i+1}}{2} \right) s_{i+1} = 0 \quad (4.5)\]

4.2.2.1.3. Boundary Condition at the Crack Face

The representative nodes “D” in Fig. 4.5(c) shows the force boundary condition at the right end node \( n \) of the left segment, which can be expressed as

\[ P - F_{n-\frac{1}{2}} - B_n = 0 \]
where \( F_{n-1/2} \) is the yarn force at a distance \(-h/2\) from the end node. The force can be calculated by using the yarn area \( A_y \), average secant modulus \( E_y \) at node \( n-1 \) and \( n \) and the yarn strain at node \( n \), which is approximated by backward difference method

\[
F_{n-1/2} = A_y \left( \frac{E_{y,n-1} + E_{y,n}}{2} \right) \left( s_n - s_{n-1} \right) \frac{h}{2} \quad (4.7)
\]

At node \( n \), bond stress \( \tau_n \) is determined by the secant modulus \( k_n \) and slip \( s_n \). The resistance bond force \( B_n \) is obtained by integrating the bond stress over the surface area of the yarn defined by the perimeter \( \psi \) and a half spacing distance \( h/2 \).

\[
B_n = \frac{1}{2} \psi k_n s_n h \quad (4.8)
\]

Substitute Eqs. (4.7) and (4.8) into Eq. (4.6), and rearrange the terms to obtain the equilibrium for node “D” as

\[
A_y \left( \frac{E_{y,n-1} + E_{y,n}}{2} \right) s_{n-1} - \left[ A_y \left( \frac{E_{y,n-1} + E_{y,n}}{2} \right) + \frac{1}{2} \psi k_n h^2 \right] s_n = -Ph \quad (4.9)
\]

In the reverse direction, the force equilibrium for the representative node “E” at the first node of the middle segment is

\[
-P + F_{1+1/2} - B_1 = 0 \quad (4.10)
\]

It is noted that the term \(-B_1\) in Eq. (4.10) is positive to the right since the slip distribution in the left half of the middle segment is negative [4.5(b)]. The yarn force at a distance \(+h/2\) from node 1, \( F_{1+1/2} \), and the bond force at node 1, \( B_1 \), can be expressed as
Substitute Eqs. (4.11) and (4.12) into Eq. (4.10), and rearrange the terms, the equilibrium equation for the node “E” is obtained as

\[
\begin{bmatrix}
A_y \left( \frac{E_{y1} + E_{y2}}{2} \right) + \frac{1}{2} \psi k_1 s_1 h^2 \\
-\frac{A_y}{2} \left( \frac{E_{y1} + E_{y2}}{2} \right) s_2
\end{bmatrix} = -P h
\]

The known term, \( P h \), on the right hand side of Eq. (4.9) and (4.13) forms the driving force vector \( \{T\} \) in assembling of the system of equations.

**4.2.2.1.4. Boundary Condition at the Right End Node of the Last Segment**

Finally, the boundary condition at the right end node \( n \) of the last segment as shown with labeled “F” in Fig. 4.5(c) can be written as

\[
F_{n-\frac{1}{2}} - B_n = 0
\]

Again, the value of \(-B_n\) is positive to the right in the free body diagram due to the negative slip at node \( n \). The yarn force at a distance \(-h/2\) from node \( n \), \( F_{n-1/2} \), and the bond force at node \( n \), \( B_n \), can be expressed as

\[
F_{n-\frac{1}{2}} = A_y \left( \frac{E_{y,n-1} + E_{y,n}}{2} \right) \left( s_n - s_{n-1} \right)
\]

\[
B_n = \frac{1}{2} \psi k_n s_n h
\]
\[-A_y \frac{E_{y,n-1} + E_{y,n}}{2} s_{n-1} + \left[ A_y \frac{E_{y,n-1} + E_{y,n}}{2} + \frac{1}{2} \psi k_n h^2 \right] s_n = 0 \] (4.17)

4.2.2.2. Matrix Assemblage

As can be seen from Fig. 4.5(a), each pullout segment in a cracked specimen can be solved independently by subdividing the length \(L_{x}^{(j)}\) in to \(n\) number of local nodes and applying appropriate equilibrium Eqs. (4.3), (4.4), (4.5), (4.9), (4.13) and (4.17) for each typical node (“A”–“F”) in the segment to form a set of algebraic equations which consists of a coefficient matrix \([C]\), unknown nodal slip vector \{S\} and the known driving force vector \{T\} as follows:

\[ [C]_{n,n} \{S\}_{n} = \{T\}_{n} \] (4.18)

It should be pointed out that the tension stiffening model is applicable for load control only; thus, the impose end slip \(s^*\) according to displacement control presented in the previous chapter is ignored in the matrix assemblage.

4.2.2.3. Material Stiffness Updating and Convergence Check

The same material stiffness updating algorithm and convergence check described in the previous chapter (Section 3.3.2.3 and Fig. 3.20) is used in the tension stiffening model presented in this chapter. For each load step \(j\) in the postcrack state, the system of equations for each pullout segment is assembled and solved for the nodal slips, which are then used to calculate nodal yarn strain \(\varepsilon_y\) defined by Eq. (4.2) using forward, central and backward finite difference for node 1, 1 to \(n-1\) and \(n\), respectively. The current secant modulus \(k^{(j)}\), \(E_{y}^{(j)}\) and \(g^{(j)}\) which are a function of slip and yarn strain in the free form.
models are updated at each node. If all updated modulus are very close to those in the previous step within a specified tolerance, the convergence is accepted and the next load step is applied. Otherwise, the new modulus are estimated from the current and the previous values and to form the next system of equations and solve for slip until convergence is achieved.

4.2.3. Algorithm for Simulation of Tensile Responses under Load Control Test

A tension specimen is modeled by a representative volume which has the same cross sectional area of the composite and the same number of longitudinal yarns. Bond characteristic at interface, stress strain behavior of longitudinal yarn and optional nonlinear spring simulating cross yarn junctions can be defined by free form functions. The simulation procedures can be summarized as followed:

1) The total length of specimen is discretized into $N$ number of nodes with an equal spacing of $h$. Optional springs can be attached to the junction nodes at equal transverse yarn spacing. The total nodes are divided into two groups: nodes in the end grip zones and nodes within the clear length $L_c$. They all functions the same except that the nodes in the end grips are not allowed to crack since the end zones are strengthened with metal plates while the nodes in the clear length are allowed to crack;

2) Any probabilistic distribution functions can be used to generate uniform or random matrix strength at nodes along the specimen length to achieve deterministic or stochastic crack pattern;
3) For the applied load less than cracking limit, the tensile responses of the composite are calculated by the rule of mixture and strain compatibility;

4) Once cracking, the specimen breaks into number of segments. Each segment is modeled as a pullout problem and solved iteratively for each load step. Finally, the solutions of all segments are combined to represent the responses of a whole tension specimen;

5) The analysis is stopped when the stress in yarn reaches its ultimate tensile strength or no solution is found at a specified load step due to instability of slip (very large slip value) at end grip is detected in load control test; and

6) By ignoring small matrix strain as stated in the assumption, material responses of tension specimen can be calculated as the followings:

   a. Tensile force in matrix $M(x)$ is obtained by subtracting the applied load $T$ with the force in longitudinal yarn $F(x)$. The matrix stress $\sigma_m(x)$ is obtained by dividing the force $M(x)$ with its cross sectional area $A_m$;

   b. The averaged matrix stress $\sigma_{m,avg}$ across several crack spacing $C_{spac}$ is obtained by averaging the matrix stress $\sigma_m(x)$ at all nodes within the clear length $L_c$. The averaged matrix strain $\varepsilon_{m,avg}$ is obtained by dividing the averaged matrix stress $\sigma_{m,avg}$ with the Young’s modulus of matrix $E_m$. Elastic elongation of the matrix $\delta_m$ is obtained by multiplying the average matrix strain $\varepsilon_{m,avg}$ to the clear length $L_c$;

   c. Crack width $w_{cr}$ at each crack station is calculated by adding the absolute values of slip at the right and the left of a crack face;
d. Composite strain $\varepsilon_c$ is obtained by dividing the total elongation with the clear length $L_c$, in which the total elongation is calculated by summing crack width $w_{cr}$ at all crack stations and the elastic elongation of the matrix $\delta_m$;

e. Composite stress $\sigma_c$ is obtained by dividing the applied load $T$ with the composite sectional area $A_c$; and

f. Crack spacing $C_{spac}$ is obtained directly from the length of crack segments within the clear length $L_c$.

**4.2.4. Parametric Studies of Material Parameters**

The four main factors including bond-slip relationship, matrix strength, levels of slack in longitudinal yarn and mechanical anchorage provided at cross yarn junctions which affect the tensile performance of fabric reinforced concrete were studied. The magnitude and shape of material models are chosen in a sensitive range, which are not intended for any particular reinforced composite systems.

**4.2.4.1. Mechanical Responses of a Numerical Base Model**

A numerical base model as shown in Fig. 4.7 was first simulated and its material responses would be explained and later used as references in the next study. The model was a representative volume of a composite containing only one longitudinal yarn with a diameter of 0.5 mm inside a cross sectional area of 19.63 mm$^2$, which resulted in 1% volume fraction. The total length, end length and clear length were 300 mm, 50 mm and 200 mm, respectively. Young’s modulus of the matrix was 20,000 MPa. Three material models at base levels shown in Fig. 4.6(a-c) $\tau_{\text{max}\_\text{base}} = 2$ MPa for bond, $\sigma_{m,\text{cr}\_\text{base}} = 2$ MPa for matrix and $\varepsilon_{\text{slack}\_\text{base}} = 0$ (no slack) for yarn were used. The ultimate yarn strength $\sigma_{yu}$
was set to 750 MPa to terminate the tensile responses. Spring was not used in the first parametric study. The total length was discretized into 3001 nodes with equal spacing of 0.1 mm. In order to have a unique solution, deterministic crack pattern was chosen.

Due to the symmetry of the deterministic crack pattern, only the left half of a representative volume is plotted for material responses: matrix stress, yarn stress, slip and bond stress distribution as shown in Figs. 4.8(a-d). Six subplots for each figure represent the responses at critical load steps. The bottommost shows the maximum elastic response at uncracked state, the subsequent subplots present the responses corresponding to each new crack appears in the specimen; and the topmost shows the response at final load step. For brevity of presentation, the critical load levels are shown only once in Fig. 4.8(a) and presented as a normalized value with respect to the first cracking load \( P_{1st,cr} \) of 40.2 N.

The bottommost subplot of Fig. 4.8(a) shows the matrix stress distribution in an uncracked representative volume just before the first crack appears. Using strain compatibility concept that strain of matrix, fiber and composite are all equal, the maximum cracking load \( P_{1st,cr} \) is determined to be 40.2 N (or 1.0 as a normalized value). The next subplot shows the stress redistribution when the first crack appears at the center \( 0.5L_c \), which the normalized load remains the same at 1.0. At the crack location, the matrix stress is zero and it increases rapidly toward the left. The shape of the stress is controlled by the bond-slip relationship while the magnitude is limited to the matrix strength \( \sigma_{m,cr} \). It can be seen that at this load step the stress redistribution reaches the matrix strength; thus, the next cracks will occur right after the first crack. With a little increase of load from 1.0 to 1.01 in the third subplot, additional crack is found at the
beginning of the clear length $X/L_c = 0.0$. It should be noted that the maximum stress in the previous state is located in the grip zone but crack is not allowed in that portion as it is well reinforced with metal plates. The new stress redistribution at this cracking state also reveals that the maximum matrix stress is still close to its matrix strength and additional crack will occur soon. The fourth and fifth subplots indicate low magnitude of matrix stress distribution in the uncracked segments which implies that significant amount of load is required to cause additional cracks. In the last load step shown in the topmost subplot, number of crack remains the same while the load increases. There is no change in stress pattern as but the magnitude is increased due to the uncracked segments accumulate stress transferred from the yarn by bond mechanism.

Fig. 4.8(b) presents the tensile stress in longitudinal yarn at the uncracked stage, cracking stages and the final load step. In the bottommost subplot, the uncracked stiffness of the matrix is very large compared to the yarn’s that has volume fraction only 1%; thus, most of the force is carried by the matrix and the yarn carries less than 1% of the yarn capacity $\sigma_{yu}$. Once the first crack appears in the second subplot, the yarn stress distribution changes significantly, which stress is peak at the crack location $0.5L_c$ and decreases rapidly toward the end grip as the load carried by yarn is transferred to the matrix. The third and fourth subplots that correspond to the subsequent cracking stages show significant change in stress redistribution due to more cracks appear but little increase in the maximum yarn stress since the load is marginally increased. As can be seen in the fifth subplot, significant amount of load is required to create additional crack and results in higher maximum yarn stress. Similar to the matrix, the final load step
shows the same stress pattern but the magnitude increases and reaches the ultimate yarn strength \( \sigma_y/\sigma_{yu} = 1.0 \).

Figs. 4.8(c and d) illustrate the slip and bond stress developed at the interface of the two materials. It should be noted that the sign of slip and shear stress are positive when they move to the right and negative to the left. The bottommost subplots of Figs. 4.8(c and d) correspond to the perfect bond state, which slip and bond stress are zero. The next subplot shows the first crack breaks the specimen into two pieces; the maximum positive slip and bond stress appear at the right \((0.5X/Lc)\) and decrease rapidly toward the left. When additional crack appears in the third subplots, the slip in the left end segment moves to the right and the slip in the middle segment moves in both left and right directions. The subsequent fourth and fifth subplots show that the slip and bond stress distribution at later cracking stages. Finally, the topmost subplot shows the responses at the final load step of yarn failure.

4.2.4.2. The Effect of Four Main Factors

Parametric studies were conducted to demonstrate the sensitivity of the numerical base model due to the changes in bond strength, matrix strength, levels of slack in yarn and spring strength. Three normalized material responses including composite tensile stress, averaged matrix stress and average crack spacing as a function of composite strain were selected for the study. The same representative volume of the numerical base model was used again but at different levels of constitutive models depicted in Fig. 4.6(a-d). The combination of material models for each run is listed in Table 4.1. The underscore “\_base” refers to the necessary base material models for simulation while “\_opt” refers to
optional model parameters that can be included to the numerical base model in simulations. When springs were used in the numerical model to study the effect of restraint due to transverse yarns, the springs were spaced 5 mm apart.

Fig. 4.9 shows the composite tensile stress strain response of the numerical base model due to the change of each parameter studied. The stress and strain were normalized with matrix cracking strength (2 MPa) and strain (0.0001 mm/mm) at base level, respectively. As can be seen from Fig. 4.9(a), the increase in bond strength from 1 to 3 marginally increases the postcrack stiffness of the composites. This stiffening effect is caused by higher bond strength allows the intact concrete between two parallel cracks to carry more load such that the contribution from cracked matrix is significant. The maximum responses of these three curves are, however, limited to the same ultimate yarn failure \( \sigma_y \) of 750 MPa. Fig. 4.9(b) reveals that the increase in matrix strength directly improves the postcrack stiffness because higher force is required to break specimen. Likewise the maximum responses of three matrix strength levels are limited to the yarn failure. Fig. 4.9(c) shows the effect of slack that delay the action of longitudinal yarn to carry the applied load. As more slack allowed, more yarn extension is given up, which results in larger strain at early cracking stages. Once stress in the yarn reaches elastic range, all three slack levels yield relatively the same postcrack stiffness. Finally, Fig. 4.9(d) shows that the increase in restraining levels of spring attached to the longitudinal yarn has a positive effect in toughening postcrack stiffness. For the number of junctions and selected strength levels used in the comparison, the effects of increasing spring strength are slightly lower than the effect of increasing bond strengths.
Fig. 4.10 shows the effect of each factor to the change of normalized averaged matrix stress and composite strain. All subfigures reveal a zigzag pattern in the postcrack stage. The drop in matrix stress corresponds to the crack event that the matrix releases the force to the yarn and the increase in stress corresponds matrix accumulates stress back from the yarn by mean of bond mechanism. Fig. 4.10(a) reveals that increasing bond strength 3 times apparently has no effect to the postcrack stress. This is due to the assumed perfect brittleness of matrix, which makes the force in composite to be controlled by the force in yarn at crack location. The bond strength only controls the magnitude of zigzag response; higher bond strength higher force can be stored in matrix and also larger drop when the matrix cracks. Without inherit postcrack strength of the matrix commonly defined by stress crack width relationship, the increase in bond that minimizes crack width will not show the improvement of the postcrack stress. The brittle matrix used in the simulations has no residual stress at any levels of crack width. This finding agrees with the analytical effective tensile stress strain curve for reinforced concrete proposed by Gupta and Maestrini (1990), that the model is independent of bond parameters but a function of concrete tensile strength, yield stress of steel, modulus ratio and reinforcement ratio. Fig. 4.10(b) shows that using higher matrix strength increase peak strength as well as the postcrack responses, almost proportional to the degree of increasing of matrix strength. Fig. 4.10(c) shows that the effect of increasing slack in the yarn model results in smoother postcrack response between normalized strain 1 and 50. This due to the slack at early cracking states give more strain such that the zigzag responses become more flat; however, when stress in the yarn begins the elastic response, the postcrack responses
become zigzag again as shown in the normalized strain greater than 50. Fig. 4.10(d) shows that adding springs to the longitudinal yarn provide more resistance to slip and leads to smaller normalized strain for the same load level.

The last material response for comparison is the normalized average crack spacing at different normalized composite strain levels. Fig. 4.11(a) explains the observed crack evolution in tension specimen that the increase in bond strength leads to smaller crack spacing. This is due to the fact that higher bond can transfer more load from yarn to matrix causing more cracks or shorter crack spacing in a tension specimen. The effect of using stronger matrix [Fig. 4.11(b)] that makes the specimen more difficult to break results in longer crack spacing for the same level of strain as expected. The same final crack spacing of $0.125L_c$ for all curves are due to the ultimate load is govern by yarn failure. The effect of increasing slack is shown in Fig. 4.11(c) which appears to increase normalized strain at the same level of crack spacing for the same reasons as explained before. Fig. 4.11(d) reveals that the increase of spring strength has the same effect as increasing bond strength but at smaller magnitude for the number of springs and their strengths used in this study.

4.2.5. Simulations of Tension Members

Based on availability of bond properties and tension test data, three sets of experiments conducted independently were used in the simulations: a steel reinforced concrete system, Alkaline Resistant (AR)-glass textile reinforced concrete with an equivalent bond slip model and AR-glass fabric reinforced cement paste in which the bond slip and cross yarn junction are modeled separately.
4.2.5.1. Simulation of Reinforced Concrete Tension Members

Two samples of reinforced concrete tension specimens conducted by Mariza and Houde (1979) were chosen in the simulation. Fig. 4.12(a) shows the test set up of a 16 inch specimen reinforced with rebar #8. The specimen was pulled on both sides and the load-elongation responses were recorded. Two size of cross section 8”x8” and 6”x6” were chosen as the steel axial stiffness is several time higher than the concrete axial stiffness, such that the effect of concrete strain is negligible as the assumption used in the derivation. Note that the test setup for this case was different from the typical tension sample used in the parametric studies; i.e., no metal plate attached to the end zones. The material properties of concrete matrix were as followed: the ultimate compressive strength $f_c' = 5.94$ ksi, tensile strength $\sigma_{m,cr} = 0.343$ ksi and the Young modulus $E_m = 4.36$ ksi. The material properties of steel are: the yield stress $f_{sy} = 60$ ksi and the Young’s modulus $E_s = 28.4 \times 10^3$ ksi.

Fig 4.12(b) shows the experimental bond-slip relationship of rebar #8 derived from 14 pullout tests of concentric tension specimens and 6 pullout tests of beam end specimens (Houde 1973). The best fitted polynomial function was reported as

$$\tau = 1.95 \times 10^6 s - 2.35 \times 10^9 s^2 + 1.39 \times 10^{12} s^3 - 0.33 \times 10^{15} s^4$$

(4.19)

where $\tau$ is the bond stress in psi and $s$ is the slip in inches. The bond-slip relationships obtained at different locations of the sensors throughout the length of tension specimen were the same for all loads up to the peak (or at slip of 0.0012 in.). After the peak, the bond stress obtained from the sensors at distance greater than 3-4 inches from the end face was almost constant while the bond stress derived from the sensors close to the end
face progressively decreases with increasing of slip. The lost of bond stress near the ends was attributed to the splitting cracks observed in experiments. For the simulations in this study, Eq. (4.19) was used to generate the input bond slip model for the slip up to 0.0016 inches and then the bond stress is assumed to remain constant as shown by the solid curve in Fig. 4.12(b).

The tension members were model with 801 nodes and the uniform tensile strength of 0.343 ksi is specified for all nodes throughout the length of specimen. In order to compare the simulation with the experimental results, the applied load at each load step was divided with an area of steel rebar to obtain nominal steel stress and the steel strain distribution was integrated over the length of specimen to obtain the elongation of the tension members. The predicted steel stress elongation response for cross section 8”x8” and 6”x6” were compared with experimental results in the Fig. 4.12(c). It can be seen that the simulation of the larger section 8”x8” reasonably matched with the experimental result. In this case, there was no drop in steel stress at any steel elongation; thus, the specimen did not crack; only the slip occurs at the interface. There was no report on the crack status of this specimen in literature. The predicted response of the smaller section 6”x6” shows reasonable agreed with the experimental result. From the simulation revealed that the concrete cracked at steel stress of 18 ksi, which was very close to the test report that cracks first appear at stress level of 15-20 ksi and it propagated from inner to the outer surface at stress of 40 ksi.
4.2.5.2. Simulation of AR-Glass Fabric Reinforced Concrete with Equivalent Bond Slip Model

Uniaxial tension and pullout test results of AR-glass fabric reinforced concrete reported in literature (Jesse 2005; Richter and Zastrau 2006) were chosen to demonstrate the predictability of the proposed model based on equivalent bond model. The sketch of tensile specimen and the pullout test result are shown in Fig. 4.13(a and b). The total length of specimen was 500 mm with a clear length of 300 mm and the grip length of 100 mm. The width and thickness were 100 mm and 8 mm respectively. The sample was reinforced with three layers of AR-glass fabrics that had a yarn count of 310 tex for the total volume fraction of 1.4%. The Young’s modulus of the yarn was 76,000 MPa and its ultimate tensile strength was 1,300 MPa. The matrix used was fine aggregate concrete with an elastic modulus of 30,000 MPa and tensile strength of 7 MPa. The bond slip relationship shown in Fig. 4.13(b) was determined by yarn pullout experiments (Richter and Zastrau 2006). The yarn cross sectional area of 0.11 mm$^2$ was taken from literature (Haussler-Combe and Hartig 2007), which used the same fabric (yarn count of 310 tex). Assuming circular shape, each yarn had a diameter of 0.374 mm and perimeter of 1.17 mm. The total number of yarns according to the volume fraction 1.4% was 102 and total perimeter was 119.8 mm.

The simulation was run using a nonlinear bond model as shown in Fig. 4.13(b), uniform matrix strength of 7 MPa and a yarn stress strain model without slack. Deterministic crack pattern as done before in parametric study was used again in the simulation and the result was compared with an averaged response of 5 samples.
Fig. 4.14(a) shows the comparison between the simulation and the averaged experimental response. The proposed model predicted well for most part but overestimate the ultimate capacity at the end. Considering the probability of failure that all 102 yarns in the specimen may not break at the same ultimate tensile strength of 1300 MPa but the weakest one will fail first and the load will be redistributed to the remaining yarns, leading to progressive failure. With this hypothesis, if the weakest yarn had strength of 892 MPa as marked with “X” symbol in Fig 4.14(a), the simulation would match with experimental result. For comparison purpose, the simulation using analytical approach by Ritcher et al. (2006) was also plotted in the same figure. The prediction match well with their experimental data for entire range but the predicted response was fluctuated at early postcrack stage which has not been observed in experiments.

With this proposed tension stiffening model, the force contribution from yarn and matrix can be analyzed as shown in Fig. 4.14(b). It shows that concrete contribute most of the strength in the uncracked stage while the fabric play an important role after the concrete cracks. Superposition of these two forces results in the total composite load. With good bonding at the interface, cracked concrete can hold some tensile force and provide additional stiffness to the postcrack response. The force in cracked concrete is divided by its cross sectional area to yield concrete stress and plotted at different level of composite strain to represent tension stiffening response as shown in Fig. 4.14(c). It shows that the peak strength is equal to the matrix strength specified at 7 MPa and drops sharply to 3 MPa during crack evolution and continues to drop gradually to 2.6 MPa after no more cracks occur. The last Fig. 4.14(d) represents the averaged crack spacing and
crack width evolution as function of composite strain. It can be seen that the crack spacing decrease rapidly in early crack development and the spacing become constant of the length 18.75 mm at strain of 0.005 mm/mm. The corresponding crack width starts at zero in elastic stage, and suddenly increases to 0.078 mm at the first crack appeared. Then the crack width remains relatively constant during early crack development. After number of cracks is stabilized at composite strain of 0.005, the width starts to increase rapidly reaching 0.3 mm.

4.2.5.3. Simulation of AR-Glass Fabric Reinforced Concrete with Bond-Slip and Spring Model

An AR-glass fabric reinforced cement composite as shown in Fig. 4.1(a) conducted by Pahilajani (2004) was used in the simulation. Three replicates were tested and the sample that yielded a median response was the target in the simulations. The specimen had a total length, end grips and clear span of 180 mm, 30 mm and 120 mm, respectively. The width and thickness were 22.7 mm and 12.5 mm respectively. Eight layers of AR-glass fabric were used and each fabric contains 5 pairs of yarns per inch in both longitudinal and transverse directions; this results in 72 yarns for a cross section and the spacing of cross yarn junctions was 5.10 mm. The yarn had an approximate equivalent diameter of 0.27 mm with the ultimate tensile strength of 1260 MPa. The matrix used was a plain cement paste. The composite modulus was 1,456 MPa and its first cracking strength and strain were 3.49 MPa and 0.0024 mm/mm, respectively. The load capacity of a cross yarn junction was 9 N obtained from the test of plain fabric (Sueki 2003).
Since there was no fabric pullout test result for this tension specimen, the bond and spring models were calibrated from the closest pullout test result of the same fabric in plain cement paste (Sueki 2007). Fig. 4.15(a) shows the calibrated bond and spring models that yield the best fitted simulation to the pullout experiment as shown in Fig. 4.15(b). It should be noted that the obtained spring model plays an important role at small slip level (0 - 1.9 mm) while the bond at interface continuously provide resistant for all slip levels. The Young modulus of the cement paste was not known; thus, it was estimated from the rule of mixture of the composite at the initial elastic load by the following expression

\[
E_m = \left( E_c - \eta E_y A_y \right) \frac{A_c}{A_m}
\]  

(4.20)

where subscripts \(m\), \(y\), and \(c\) refer to matrix, yarn and composite, respectively; \(\eta\) is the axial stiffness efficiency factor defined in the previous chapter (Section 3.2.4). As several layers of fabric were used in a thin specimen, it had been observed that the late postcrack response of the composite was lower than the combined stiffness of individual fabric layer. Thus, the efficiency of fabric should be less than one to match the experimental observation. This may be attributed to the mechanics of bond at concrete - yarn sleeve interface and bond at each sub layer within a bundle. The complexity at this micro mechanics level was by pass by using a single efficiency factor \(\eta\) to the Young’s modulus of the yarn. This would make the yarn has more elongation for the same level of loading which corresponds to fewer filaments at the outer layer are effective and those at the inner layer are less effective. The tensile strength of cement paste matrix \(\sigma_{m,cr}\) was
also unknown and was obtained by back calculation from the strain at first cracking of the composite $\sigma_{c,cr}$ using rule of mixture

$$\sigma_{m,cr} = \left( \varepsilon_{c,cr}E_c - \frac{\eta\varepsilon_{c,cr}E_yA_y}{A_c} \right) \frac{A_c}{A_m}$$ (4.21)

Fig. 4.15(c) shows the experimental composite tensile stress strain responses of the three samples (circle symbol) and four simulation responses at different efficiency levels (line plots). It can be seen that if the yarn has an ideal 100% effectiveness, the response will be very stiff; in this case, it slightly over-estimates the response of the strongest sample. When decrease the efficiency to 55%, the simulation reasonable agrees with the median sample. For the efficiency level between 35% and 75%, the simulations appear to cover the range of all three samples. Fig. 4.15(d) compares evolution of crack spacing between the simulation and the median sample. The simulated crack spacing reveals that the deterministic crack pattern yields smaller crack spacing than the experimental result; however, the prediction is still within the range of the mean response minus one standard deviation. It can be tentatively concluded that the simulation using both bond and spring models can explain the general behavior of tensile response of fabric reinforced cement.

4.4. Chapter Conclusions

The proposed numerical model based on nonlinear finite difference method is a universal tool that can be used to simulate various fabrics reinforced cementitious composites under uniaxial tension test. The free form material model concept provides a great flexibility to describe any characteristics at material interface, behavior of the longitudinal yarn and mechanical anchorage provided by cross yarn junctions. The
parametric studies reveal that the increase in strength of the bond and spring models slightly improves the postcrack stiffness of the tensile response but does not discern in the tension stiffening effect. This is due to the assumption that matrix is brittle and the force in composite is limited to the force in yarn at crack location. The increase in bond and spring strength allows more force to be stored as well as the force to be released between yarn and matrix at different stages of cracking. The effect of increasing matrix strength increases the postcrack stiffness and also delay crack evolution. Finally, the increase in amount of slack in composite yields higher strain at early cracking stage; nevertheless, the late postcrack stiffness is unaffected.

The proposed numerical tension stiffening model based on non linear finite difference method was used in simulations of three independent experiments: steel reinforced concrete tension member, AR-glass fabric reinforced concrete and AR-glass fabric reinforced cement paste. For the first two experiments, the simulations used the bond slip relationship given in the literatures to predict the experimental tensile stress strain response of the composites. The predicted responses agreed well with the experimental results. The last experiment was a research attempt to explain a more realistic tensile behavior of the fabric reinforced cement that the transverse yarn in the fabric play a significant role in pullout resistant. The bond slip and spring model was derived from the closest available pullout test data and used in the simulations of the tension specimens. The simulation revealed that it was necessary to use efficiency factor to reduce the apparent Young’s modulus of the yarn in order to match with the experimental result. The efficiency was used instead of micro mechanics to explain how the filaments in a
yarn buddle contribute to the apparent yarn stiffness. A more methodical experimental program is required to identify the appropriate value of the efficiency factor for each fabric system.

**Notation**

*The following symbols are used in this paper:*

- \( A \) = cross sectional area;
- \( B \) = bond resistance force;
- \([C]\) = coefficient matrix in system of equations;
- \( C_{spac} \) = crack spacing;
- \( dx \) = finite length of yarn in matrix;
- \( E \) = Young’s modulus;
- \( F \) = force in yarn;
- \( f_c' \) = uniaxial concrete compressive strength;
- \( f_{sy} \) = yield stress of steel;
- \( G \) = spring force;
- \( g \) = secant modulus of spring force-slip model;
- \( h \) = finite length of yarn between two adjacent nodes;
- \( k \) = secant modulus of bond-slip model;
- \( L \) = total embedded length;
- \( L_c \) = clear length between two end grips in tension specimen;
- \( L_{si}^{(i)} \) = \( i^{th} \) cracked segment in tension specimen;
- \( M \) = force in matrix;
\( N \) = number of total nodes used in tension-stiffening model;
\( n \) = number of node used in each pullout segment;
\( P \) = pullout load;
\( T \) = tension load;
\( \{T\} \) = nodal tension load vector;
\( SF_{\text{max, opt}} \) = optional spring model that can be attached to the base numerical model;
\( s \) = slip at matrix-yarn interface;
\( \{S\} \) = nodal slip vector;
\( s' \) = derivative of slip with respect to longitudinal axis \( x \);
\( X \) = horizontal coordinate in embedded length;
\( w_{cr} \) = crack width;
\( \delta_m \) = matrix elongation;
\( \varepsilon \) = strain;
\( \varepsilon_{c,cr} \) = composite strain at cracking;
\( \varepsilon_{m,cr} \) = matrix strain at cracking;
\( \varepsilon_{m,avg} \) = average matrix strain;
\( \varepsilon_{\text{slack, opt}} \) = optional slack parameter at beginning of the yarn stress strain model;
\( \phi \) = diameter;
\( \eta \) = axial stiffness efficiency factor;
\( \sigma_{c,cr} \) = composite stress at cracking;
\( \sigma_{m,cr} \) = matrix stress at cracking;
\[ \sigma_{m,\text{avg}} = \text{average matrix stress}; \]
\[ \sigma_{yu} = \text{ultimate tensile strength of yarn}; \]
\[ \tau = \text{shear stress at yarn matrix interface}; \]
\[ \tau_{\text{max base}} = \text{base bond-slip model used in numerical simulation}; \]
\[ \psi = \text{equivalent circumference of yarn}. \]

**Subscripts**

\[ 1, 2, 3.. \] = numbering in numerical algorithm;
\[ c, r \] = at cracking;
\[ c, m, s, y \] = composite, matrix, steel, yarn;
\[ i + 1/2 \] = at half spacing to the right of node \( i \); and
\[ i - 1/2 \] = at half spacing to the left of node \( i \).
### Table 4.1. Combination of Material Models Used in the Parametric Studies

<table>
<thead>
<tr>
<th>Combination</th>
<th>Bond Model $\tau_{\text{max}}$ (MPa)</th>
<th>Matrix Strength $\sigma_{m,cr}$ (MPa)</th>
<th>Yarn Model $\epsilon_{\text{slack}}$ (mm/mm)</th>
<th>Spring Model $SF_{\text{max}}$ (N)</th>
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<td>Model_base</td>
<td>2.0</td>
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<td>2.0</td>
<td>0.000</td>
<td>-</td>
</tr>
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<td>2.0</td>
<td>0.000</td>
<td>-</td>
</tr>
<tr>
<td>$1.25 \times \sigma_{m,cr \text{ base}}$</td>
<td>2.0</td>
<td>2.5</td>
<td>0.000</td>
<td>-</td>
</tr>
<tr>
<td>$1.50 \times \sigma_{m,cr \text{ base}}$</td>
<td>2.0</td>
<td>3.0</td>
<td>0.000</td>
<td>-</td>
</tr>
<tr>
<td>$+1 \times \epsilon_{\text{slack opt}}$</td>
<td>2.0</td>
<td>2.0</td>
<td>$+0.001$</td>
<td>-</td>
</tr>
<tr>
<td>$+2 \times \epsilon_{\text{slack opt}}$</td>
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<td>2.0</td>
<td>$+0.002$</td>
<td>-</td>
</tr>
<tr>
<td>$+3 \times \epsilon_{\text{slack opt}}$</td>
<td>2.0</td>
<td>2.0</td>
<td>$+0.003$</td>
<td>-</td>
</tr>
<tr>
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<td>$+6.0$</td>
</tr>
<tr>
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<td>2.0</td>
<td>0.000</td>
<td>$+9.0$</td>
</tr>
</tbody>
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Fig. 4.1. Typical tension specimens made of four types of fabrics: (a) AR-glass; (b) Polypropylene (PP); (c) Polyvinyl Alcohol (PVA); (d) Polyethylene
Fig. 4.2. Typical tensile response of fabric reinforced cement based composites
Fig. 4.3. Mechanical behavior of a cracked fabric reinforced cement composite; (a) bond slip model; (b) longitudinal yarn stress strain model; (c) nonlinear spring model simulating anchorage strength at cross yarn junction; (d) matrix strength for cracking criterion
Fig. 4.4. Matrix strength distribution in tension specimen to achieve different crack patterns; (a) deterministic crack; (b) stochastic crack
(a) Total $N$ number of nodes with equal spacing of $h$

(b) $F_n = P$

$F_1 = 0$

$s(\cdot)$

$s(-\cdot)$
Fig. 4.5. Finite difference model; (a) discretized fabric pullout model under load control (b) sign convention for slip and boundary conditions for force in yarn; (c) free body diagram of six representative nodes labeled as “A” – “F”
(a) Bond Stress (MPa) vs. Slip (mm)

- $3 \times \tau_{\text{max_base}}$
- $2 \times \tau_{\text{max_base}}$
- $1 \times \tau_{\text{max_base}}$

$\tau_{\text{max_base}} = 2$ MPa

(b) Matrix Strength (MPa) vs. Matrix Grade

- $1.25 \times \sigma_{m,\text{cr_base}}$
- $1.0 \times \sigma_{m,\text{cr_base}}$
- $1.5 \times \sigma_{m,\text{cr_base}}$

$\sigma_{m,\text{cr_base}} = 2.0$ MPa
Fig. 4.6. Material models for parametric studies (a) bond slip model; (b) matrix strength, (c) yarn stress strain model; (d) nonlinear spring model
Fig. 4.7. Numerical base model using base material models (no options)
Normalized Matrix Stress \( \left( \frac{\sigma_m}{\sigma_{m,cr}} \right) \)

- Elastic (uncracked)
  - \( P_{elas} = P_{1st,ck} = 40.2 \, \text{N} \)
- \( P'_{1st,ck} = 1.00 \)
- \( P'_{2nd,ck} = 1.01 \)
- \( P'_{3rd,ck} = 1.06 \)
- \( P'_{4th,ck} = 1.57 \)
- \( P'_{\text{final}} = 3.67 \)
Normalized Yarn Stress, $\sigma_y/\sigma_{yu}$

Normalized Distance, $X/L_c$
Fig. 4.8. Four normalized material responses of a numerical base model: (a) matrix stress distribution; (b) yarn stress distribution; (c) slip distribution; (d) shear stress distribution
Fig. 4.9. Normalized composite stress strain responses due to variation of; (a) bond strength; (b) matrix strength; (c) slack in longitudinal yarn; (d) spring strength
Fig. 4.10. Normalized averaged tensile stresses in cracked matrix as a function of normalized composite strain due to variation of; (a) bond strength; (b) matrix strength; (c) slack in longitudinal yarn; (d) spring strength
Fig. 4.11. Normalized averaged crack spacing as a function of normalized composite strain due to variation of; (a) bond strength; (b) matrix strength; (c) slack in longitudinal yarn; (d) spring strength
(a) 

\[ \text{Rebar \#8} \]

- 8"x8"
- 6"x6"

- 16"

- \( f'_{c} = 5.94 \text{ ksi} \)
- \( \sigma_{m,cr} = 0.343 \text{ ksi} \)
- \( E_{m} = 4.36 \times 10^{3} \text{ ksi} \)
- \( f_{sy} = 60 \text{ ksi} \)
- \( E_{s} = 28.4 \times 10^{3} \text{ ksi} \)

(b) 

- Bond Stress (ksi) vs. Slip (in)
- Fitted Function
- Beam Tests
- Pull Tests
Fig. 4.12. Simulation of reinforced concrete tension members; (a) test setup; (b) bond slip model; (c) steel stress vs. elongation responses
Fig. 4.13. Tension specimen of AR-glass fabric reinforced concrete and its bond slip relationship; (a) specimen; (b) bond slip model
Composite Tensile Stress (MPa)

Experiment
Proposed Model
Ritcher, M.

Composite Tensile Strain (mm/mm)

σ_f = 892 MPa

Average Force in Crack Specimen (kN)

Composite (Expr.)
Composite (Sim.)
Yarn (Sim.)
Matrix (Sim.)

Composite Tensile Strain (mm/mm)
Fig. 4.14. Simulation of AR-glass fabric reinforced concrete; (a) composite tensile stress strain (b) force components in composite; (c) tension stiffening in postcrack concrete; (d) average crack spacing and crack width as various stage of composite tensile strain.
Fig. 4.15. Simulation of AR-glass fabric reinforced cement using bond and spring models; (a) bond and spring model; (b) best fit simulation of fabric pullout response; (c) effect of efficiency factor in prediction of composite tensile stress strain response; (d) crack spacing evolution as function of composite strain.
CHAPTER 5

FLEXURAL MODELING OF FIBER REINFORCED CEMENT COMPOSITES

5.1. Introduction

Reinforcement of cementitious materials with short randomly distributed fibers has been successfully practiced for more than 40 years (Mobasher and Shah 1989; Mobasher and Li 1996a). Due to the low fracture toughness of cement-based materials, tensile cracks occur easily due to applied stress, restraint, or environmental conditions. The interfacial bond developed between the fibers and matrix utilizes the strength and stiffness of the fibers in reinforcing the brittle matrix. Once the matrix cracks, load still transfers across the crack faces through the bridging fibers. As the load on the composite is increased, the process of fiber pullout affects load carrying capacity and further contributes to energy dissipation. It has also been known when using a high volume fraction of fibers with a high specific surface area, the crack bridging potential and the strength of the composite are increased (Romualdi and Batson 1963; Majumdar and Laws 1983; Mobasher and Li 1996b). An increase in volume fraction of the fiber shows an enhancement of tensile strength in a variety of discrete fiber systems, including Steel Fiber Reinforced Concrete (SFRC) (Umekawa and Nakazawa 1970; Lim and Paramasivam 1987a,b), Glass Fiber Reinforced Concrete (GFRC) (PCI Committee 1993), Slurry Infiltrated Mat Concrete (SIMCON) (Krstulovic-Opara and Malak 1997; Bayasi and Zeng 1997), Ductal (Chanvillard and Rigaud 2003) and Engineered Cementitious Composite (ECC) (Li 1994; Maalej and Li 1994; Maalej 2004). Similar behaviors are also observed in continuous fiber systems such as Ferrocement (FRC) (Naaman and Shah 1971) and Textile Reinforced Cements (TRCs) (Peled and Mobasher 2005; Mobasher et al. 2006a,b), which
show improved tension capacity and ductility. In the continuous systems, the increase in strength is associated with distributed cracking mechanisms and strain-hardening behavior (Mobasher et al. 2006a,b).

Naaman and Reinhardt (2006) defined the formulation and testing conditions which lead to strain-hardening or strain-softening in tension, and within this last category, additional terms of deflection-hardening or deflection-softening are defined. The strain-softening region of tensile response contributes to the load-carrying capacity and nonlinear energy dissipation. When subjected to flexural stresses, the post-peak response in tensile regions contributes to load carrying capacity in the softening observed in the deflection response. However, if the volume fraction of fibers is sufficiently high, the stiffness contribution of cracked zone may result in loads in excess of the first cracking point and defined as deflection-hardening. Therefore the increased capacity in flexure at large deflection levels is due to contributions from the stiffness of cracked zone in tensile regions.

The non-linear behavior of fiber reinforced concrete (FRC) is best characterized by closed loop controlled tests conducted in tension or flexure. Fig. 5.1(a) presents tensile stress–strain response as compared to equivalently elastic flexural stress vs. deflection of Alkali Resistant (AR) glass fabric reinforced composite material (Aldea et al. 2007). The flexural stress is calculated based on the assumption that materials behave elastically throughout the uncracked and postcrack responses, resulting in equivalent stress. It is critically important to observe that the general shapes of these curves are similar as they represent initial linear portions followed by a range with a reduced stiffness that is due to
distributed cracking. There is however a fundamental difference in the magnitudes of stress from the tests and the associated deformations. The two main parameters characterizing the tensile response are the cracking tensile strength or Bend Over Point (BOP) (Shah and Ouyang 1991) and the Ultimate Tensile Strength (UTS). In the flexural loading case, the cracking is referred to as the Limit of Proportionality (LOP) and ultimate strength as the Modulus of Rupture (MOR). The fact that the MOR value may be several times higher than the ultimate tensile strength UTS can be attributed to several parameters, including the nature of data reduction, size effect, and also nature of the loading. This discrepancy has been well known in the field (Majumdar and Laws 1983). This phenomenon is best shown by comparing the cumulative probability distribution functions for the four strength parameters. The distribution of BOP and UTS in tension followed by LOP and MOR in flexure are shown in Fig. 5.1(b) using the results from thin section fabric cement composites (Aldea et al. 2007). Note, that there are fundamental over-predictions of cracking and ultimate strength in flexure as compared to the tension case by as much as 300%. These indicate that use of flexural data as fundamental material properties in the design of cement composites may be misleading and unconservative. Closed form relationships that explain such differences between the tensile and flexural strengths have been mentioned for both strain-hardening (Soranakom and Mobasher 2007a; Soranakom et al. 2006b) and strain-softening type composites (Soranakom and Mobasher 2007b).

In order to correlate tensile and flexural data for various materials, a unified closed form solution for generating moment–curvature response of homogenized materials is
presented for various FRC systems. Since the solution is derived explicitly, iterative procedures required for handling material non-linearity are not necessary; hence this method is attractive for use in an inverse analysis algorithm to back-calculate material parameters from convenient flexural tests. Furthermore, the moment–curvature response generated by closed form solution can be used as an input section property for a beam element in non-linear finite element analysis to predict flexural behavior of more complex structures. Finally, the closed form solutions of the proposed equations can be subsequently used in a simplified design procedure for cement composites.

5.2. Strain-Softening Fiber Reinforced Concrete

This section presents the simplest analytical model for predicting strain-softening FRC. Despite it covers only a class of materials but FRC of this class is widely used in the fields. Simplicity of the model is a more important aspect that the derivation delivers a practical equation to be in the design routine rather than using a numerical approach that is less familiar to many practice engineers.

5.2.1. Material Models

The generalized FRC model used in the derivation is based on the idealized model proposed by Lim et al. (1987a). Fig. 5.2 shows a uniaxial parameterized model for strain-softening FRC with two intrinsic material parameters: Elastic modulus, $E$, (equal in tension and compression) and cracking tensile strain, $\varepsilon_{cr}$. All strains in the models are normalized with respect to $\varepsilon_{cr}$. Two nondimensional parameters: normalized postpeak tensile strength $\mu$ and normalized compressive yield strain $\omega$ are also defined. The tension model in Fig. 5.2(a) has a linear relationship from the origin to the cracking strain.
\( \varepsilon_{cr} \), then remains constant at the postpeak tensile strength \( \sigma_p = \mu \varepsilon_{cr} E \). The parameter \( \mu \) represents the ratio of the postpeak tensile strength to the cracking tensile strength \( (\mu = \sigma_p / \sigma_{cr}) \) and may be a function of the fiber volume fraction, geometry, stiffness and bond. Tensile strain is limited to the ultimate tensile strain at \( \varepsilon_{tu} = \beta_{tu} \varepsilon_{cr} \). Fig. 5.2(b) describes the compression model with stress increasing linearly up to the yield strain \( \varepsilon_{cy} = \omega \varepsilon_{cr} \), and remaining perfectly plastic until the termination point at the ultimate compressive strain \( \varepsilon_{cu} = \lambda_{cu} \varepsilon_{cr} \). Since the modulus in tension and compression are equal, the normalized compressive yield strain \( \omega \) can be thought as the ratio of compressive plastic strength to cracking tensile strength \( (\omega = \varepsilon_{cy} / \varepsilon_{cr} = \sigma_{cy} / \sigma_{cr}) \). The nondimensional strain measures \( \beta_{tu} \) and \( \lambda_{cu} \) are defined as the limits for terminating calculation of moment-curvature response. Both the tension and compression models can be expressed in term of functions as

\[
\sigma_t (\varepsilon_t) = \begin{cases} 
\varepsilon_t E & \text{for } 0 \leq \varepsilon_t \leq \varepsilon_{cr} \\
\mu \varepsilon_{cr} E & \text{for } \varepsilon_{cr} < \varepsilon_t \leq \varepsilon_{tu} \\
0 & \text{for } \varepsilon_t > \varepsilon_{tu}
\end{cases} \quad (5.1)
\]

\[
\sigma_c (\varepsilon_c) = \begin{cases} 
\varepsilon_c E & \text{for } 0 \leq \varepsilon_c \leq \varepsilon_{cy} \\
\omega \varepsilon_{cr} E & \text{for } \varepsilon_{cy} < \varepsilon_c \leq \varepsilon_{cu} \\
0 & \text{for } \varepsilon_c > \varepsilon_{cu}
\end{cases} \quad (5.2)
\]

5.2.2. Derivation of Moment-Curvature Response

Moment-curvature diagram of a rectangular beam, width \( b \) and depth \( h \), is derived using a classical beam theory that ignores the shear deformation. The linear strain distribution across the depth is assumed. The normalized compressive strain at the top fiber \( \lambda = \varepsilon_{ctop} / \varepsilon_{cr} \) is incrementally imposed in three stages: \( (0 < \lambda \leq 1) \), \( (1 < \lambda \leq \omega) \), and \( (\lambda > \omega) \),
and the stress strain diagrams in Figs. 5.3(a–c) are obtained from Eqs. (5.1) and (5.2). The internal force in each sub compression and tension zone is obtained from the area of stress diagram, and the normalized form with respect to the cracking force \( bhE\epsilon_{cr} \) is presented in Table 5.1. Similarly, moment-arm measured from the neutral axis to the center of each force component is also presented as a normalized form in Table 5.1. Table 5.2 shows the steps in the determination of net section force, moment, and curvature at each stage of the normalized top compressive strain \( \lambda \). The net force is obtained as the difference of the tension and compression forces, equated to zero for internal equilibrium, and solved for the neutral axis depth ratio \( k \). Moment expression is obtained by taking the force components about the neutral axis; and the curvature is determined by dividing the top compressive strain \( \epsilon_{ctop} = \lambda\epsilon_{cr} \) by the length of the neutral axis \( kh \). The moment and curvature at any strain level \((M, \phi)\) are normalized with respect to their respective values at cracking \((M_{cr}, \phi_{cr})\) and expressed in normalized forms \((M', \phi')\)

\[
M = M' M_{cr} \quad M_{cr} = \frac{1}{6} bh^2 E\epsilon_{cr} \tag{5.3}
\]

\[
\phi = \phi' \phi_{cr} \quad \phi_{cr} = \frac{2\epsilon_{cr}}{h} \tag{5.4}
\]

Table 5.3 summarizes the closed-form solutions for \( k, M' \) and \( \phi' \) as these terms refer to dimensionless quantities representing the normalized neutral axis depth, moment, and curvature for the three stages of top compressive strains in Eqs. (5.3) and (5.4). The normalization constants \((M_{cr} \) and \( \phi_{cr})\) are the values computed at tensile cracking
(ε_{top} = ε_{cr} or λ = 1). For example, the cracking moment can be represented as 

\[ M(ε_{cr}) = M'(1)M_{cr} = M_{cr}, \text{ since } M'(1) = 1. \]

In the calculation of the moment-curvature diagram, the compression or tensile mode of failure is determined by the compressive strain limit \( λ_u \). The limit of the flexural capacity is defined as the minimum of the top compressive strain limit \( λ_{cu} \) and the top compressive strain corresponding to the tensile strain limit \( λ_{tu} \)

\[ λ_u = \min(λ_{cu}, λ_{tu}) \quad (5.5) \]

The limit \( λ_{tu} \) may fall in either range 2 (\( 1 < λ_{tu} \leq ω \)) or range 3 (\( λ_{tu} > ω \)). From the linear strain diagram in Figs. 5.3(b and c), the relationship between \( λ_{tu} \) and \( β_{tu} \) at a balanced condition can be written as

\[ \frac{λ_{tu}ε_{cr}}{kh} = \beta_{tu} \frac{ε_{cr}}{h - kh} \quad (5.6) \]

Using the balanced condition and input parameters of \( λ_{tu} \) and \( β_{tu} \), Eq. (5.6) is solved in conjunction with the neutral axis depth ratio \( k \) defined in Table 5.3 for two possible ranges 2 or 3; this results in the two possibilities of \( λ_{tu} \)

\[ λ_{tu} = \begin{cases} 
2μβ_{tu} - 2μ + 1 & \text{for } β_{tu} \leq β_{crit} \\
2μβ_{tu} - 2μ + ω^2 + 1 & \text{for } β_{tu} > β_{crit} 
\end{cases} \quad (5.7) \]

where the normalized critical tensile strain is defined as \( β_{crit} = (ω^2 + 2μ - 1)/2μ \). Eq. (5.7) indicates that when \( β_{tu} \leq β_{crit} \), the calculated parameter \( λ_{tu} \) will be in range 2 (\( 1 < λ_{tu} \leq ω \)), and when \( β_{tu} > β_{crit} \), \( λ_{tu} \) will be in range 3 (\( λ_{tu} > ω \)). Finally, the smaller value of \( λ_{cu} \) and \( λ_{tu} \)
is used in Eq. (5.3) and (5.5), and Table 5.3 to determine the ultimate moment $M_u$ and curvature $\phi_u$.

**5.2.3. Moment-Curvature Response at Limit State**

From derivations given in the previous section, the ultimate moment $M_u$ commonly used as an index for the flexural strength can be easily obtained by Eqs. (5.3) and (5.5), and Table 5.3. For a typical FRC that has a sufficiently large postpeak tensile strain capacity $\beta_{tu}$, the moment behaves in a ductile manner and approaches a limit value $M_\infty$ as $\lambda$ approaches $\infty$ [Figs. 5.4(a and b)]. It should be noted that, for strain compatibility as shown in Fig. 5.3, the infinite value of $\lambda$ at the top fiber implies infinite value of $\beta$ at the bottom. For simplicity of estimation, the flexural strength $M_u$ of ductile FRC can be reasonably approximated by $M_\infty$. Normalized moment and curvature at the infinite top compressive strain level $M'_\infty$ and $\phi'_\infty$ are found by substituting $\lambda = \infty$ in the expression for $k$ in Table 5.3 to obtain $k_\infty$, and its subsequent substitutions in the expression for normalized moment and curvature in the same Table 5.3 obtains

$$k_\infty = \frac{\mu}{\omega + \mu} \quad \text{(5.8)}$$

$$M'_\infty = \frac{3\mu\omega}{\omega + \mu} \quad \text{(5.9)}$$

$$\phi'_\infty = \infty \quad \text{(5.10)}$$

Eq. (5.9) presents the normalized moment capacity $M'_\infty$ as a function of $\mu$ and $\omega$. It can be seen that the increase in normalized compressive yield strain $\omega$ in brittle materials ($\mu \approx 0$) negligibly affects the normalized moment capacity $M'_\infty$; however, an increase in
For the ductile materials ($\mu \approx 1$) significantly enhances the capacity. For an elastic perfectly plastic material with equal tension and compression yield strengths ($\mu = \omega = 1$), Eq. (5.9) validates the theoretical value that the plastic moment capacity of a rectangular section is 1.5 times its elastic yield strength (Salmon and Johnson 1990).

To obtain the critical parameter that determine the transition from a deflection-softening to a deflection-hardening material, the normalized moment capacity approximated by $M'_{\infty}$ [Eq. (5.9)] is set to the normalized cracking moment ($M'_{cr}=1$), and solve for the critical value of normalized postpeak tensile strength, $\mu_{crit}$

$$\mu_{crit} = \frac{\omega}{3\omega - 1}$$  \hspace{1cm} (5.11)

For a typical FRC with $\omega$ between 6 and 12, $\mu_{crit}$ varies in a narrow range between 0.353 and 0.343. This indicates that the postpeak tensile strength in a material must be at least 35% of its cracking strength before it can exhibit deflection-hardening.

5.2.4. Simplified Moment-Curvature Response

According to material classifications proposed by Naaman and Reinhardt 2006, as $\mu$ increases exceeding $\mu_{crit}$, the response of the beam shifts from deflection-softening to hardening as described by the solid curves in Figs. 5.4(a and b). Further simplification to the bilinear moment-curvature response [dash lines in Figs. 5.4(a and b)] allows closed-form solutions for the midspan deflection of a beam under three- and four-point bending to be formulated. The simplification is done in terms of the normalized moment and curvature that are independent of geometry ($b$ and $h$) and material properties ($E$ and $\varepsilon_{cr}$).
For a deflection-hardening material, the moment-curvature expression is simplified to a bilinear elastic postcrack response [dashed lines in Fig. 5.4(a)]. An optimization algorithm is used to find the best straight line drawn from the fixed point \((\phi_u', M_u')\) to the initial elastic curve with a minimum residual error. The intersection point \((\phi_{it}', M_{it}')\) between the elastic range and the postcrack range is redefined as the normalized bilinear cracking point \((\phi_{bcr}', M_{bcr}')\). This value is slightly higher than the original cracking point \((\phi_{cr}', M_{cr}')\).

The deflection-softening shown as a solid line in Fig. 5.4(b) is observed in samples with low postpeak tensile strength. In this case, the linearized postcrack line intersects the initial elastic response at \((\phi_{it}', M_{it}')\) which represents values lower than the cracking point \((\phi_{cr}', M_{cr}')\). Two transition points are introduced for the bilinear representation of the response. The first point is at the original cracking point \((\phi_{bcr}' = \phi_{cr}', M_{bcr}' = M_{cr}')\) as shown in Fig. 5.4(b). The second point is at the same location of the normalized bilinear cracking curvature \(\phi_{bcr}\) but the moment drops from the original strength \(M_{cr}'\) to the normalized reduced cracking moment \(M_{cr2}'\). This value is greater than the normalized moment \(M_{it}'\) at the intersection. Therefore, an expression for \(M_{cr2}'\) at \(\phi_{bcr}'\) can be obtained by using a linear interpolation between \(\phi_{it}'\) and \(\phi_u'\).

\[
M_{cr2}' = (1 - \psi) M_{it}' + \psi M_u' \quad \text{where} \quad \psi = \frac{(\phi_{bcr}' - \phi_{it}')}{(\phi_u' - \phi_{it}')} \tag{5.12}
\]

An alternative method in finding the intersection point is to use a regression equation established from the preliminary results using the optimization for a practical range of
parameters \( \omega, \varepsilon_{tu} \) and \( \mu \) as the intersection point is sensitive to these three parameters. Each parameter is varied one value at a time from low to high as shown in Fig. 5.5(a) while other less sensitive parameters \( b, h, E, \varepsilon_{cr}, \) and \( \varepsilon_{cu} \) are fixed. The total of 63 combinations of variables (3x3x7) was used to generate the normalized moment-curvature diagrams, intersection points and their approximate bilinear models. It is observed that the normalized postpeak tensile strength \( \mu \) is the most sensitive parameter affecting \( M'_{it} \) and \( M'_{us} \), while the other two parameters, \( \varepsilon_{tu} \) and \( \omega \), are much less sensitive. Thus, only parameter \( \mu \) is sufficient as an independent variable in the linear regression equation to predict \( M'_{it} \). A better alternative variable is \( M'_{us} \), since it includes the effects of all three parameters \( (\omega, \varepsilon_{tu} \) and \( \mu) \) and also has a relationship to \( M'_{it} \). The regression analysis indicates that \( M'_{us} \) correlates better with the data than using \( \mu \). Therefore, the regression is given in forms

\[
M'_{it} = 0.7425M'_{us} + 0.1739 \quad \text{and} \quad \phi'_{it} = M'_{it} \quad \text{(5.13)}
\]

It should be noted that the regression Eq. (5.13) is expressed in a normalized form, free of the unit system used and \( \phi'_{it} \) always equals to \( M'_{it} \) as they are in the normalized linear elastic response [Figs. 5.4(a and b)]. Parametric analysis results of 63 combinations of variables were used in establishing a regression equation with the circle symbol in Fig. 5.5(a) showing the correlation of Eq. (5.13) between \( M'_{it} \) and \( M'_{us} \). As discussed in the previous section, \( M'_{us} \) can be approximated by \( M'_{\infty} \), the triangular symbol in Fig. 5.5(b) reveals that \( M'_{\infty} \) slightly overestimates the actual capacity \( M'_{us} \) by an average of 2.7%
with the maximum error of 8.5%. It is, thus, justifiable to use $0.9M'_{\infty}$ as a conservative estimate of flexural capacity of a beam.

### 5.2.5. Simplified Closed-Form Solutions for Load-Deflection Responses

By applying the moment-area method to the bilinear moment-curvature diagrams, midspan deflection of three- and four-point bending tests can be derived explicitly. Figs. 5.6 (a and b) show the three- and four-point bending tests and the moment distribution obtained by static equilibrium. The three patterns of curvature distribution shown in Figs. 5.6(c – e) are needed in the derivations for the midspan deflection. Fig. 5.6(c) shows the curvature distribution at the bilinear cracking $\phi_{bcr}$. After cracking, the curvature distribution depends on the normalized postpeak tensile strength $\mu$. If $\mu > \mu_{crit}$, as the postcrack curvature increases, the moment continues to increase as shown by the dash line in Fig. 5.4(a), and the corresponding curvature distribution along the beam is shown in Fig. 5.6(d). On the other hand, if $\mu < \mu_{crit}$, as the postcrack curvature increases, the moment either increases or decreases at the levels below the bilinear cracking moment $M_{bcr}$ as shown by the dash line in Fig. 5.4(b), and the corresponding curvature distribution is shown in Fig. 5.6(e). For the three-point bending that $\mu < \mu_{crit}$, the additional parameter for plastic length $L_p$ at the vicinity of the load is needed to simulate the zone undergoing localization in the postpeak response, while the nonlocalized zone is elastically unloading.

With the moment-area method, the curvature distribution of a half-model according to the internal moment distribution in Figs. 5.6(c–e) is divided into several areas. By taking the curvature areas around the left support, the midspan deflection is obtained. A set of
equations for calculating the midspan deflection $\delta$ of the three-point bending at the bilinear cracking, at ultimate when material has $\mu > \mu_{\text{crit}}$, and at ultimate when material has $\mu < \mu_{\text{crit}}$ are presented in Eqs. (5.14a)–(5.14c)

$$\delta_{bcr} = \frac{1}{12} L^2 \phi_{bcr}$$  \hspace{1cm} (5.14. a)

$$\delta_u = \frac{L^2}{24M_u^2} \left[ (2M_u^2 - M_uM_{bcr} - M_{bcr}^2) \phi_u + (M_u^2 + M_uM_{bcr}) \phi_{bcr} \right]; \hspace{1cm} \mu > \mu_{\text{crit}} \hspace{1cm} (5.14. b)$$

$$\delta_u = \frac{\phi_u L_p}{8} \left( 2L - L_p \right) + \frac{M_u \phi_{bcr}}{12M_{bcr}} \left( L - L_p \right)^2; \hspace{1cm} \mu < \mu_{\text{crit}} \hspace{1cm} (5.14. c)$$

It should be noted that Eq. 5.14(c) is slightly different from the same equation published in Soranakom and Mobasher (2007b). Slightly mistake in the elastic portion of the beam during unloading was maded in the derivation and was corrected in this dissertation. Nevertheless, the mistake was not serious and the previous published equation can be thought as a variant of the Eq. 5.14(c). They both yield almost identical result and no difference can be observed in the simulations of SFRC, which will be presented in the Section 5.2.8. For four-point bending, a set of equations can be written as

$$\delta_{bcr} = \frac{23}{216} L^2 \phi_{bcr}$$  \hspace{1cm} (5.15.a)

$$\delta_u = \frac{L^2}{216M_u^2} \left[ (23M_u^2 - 4M_uM_{bcr} - 4M_{bcr}^2) \phi_u + (4M_u^2 + 4M_uM_{bcr}) \phi_{bcr} \right]; \hspace{1cm} \mu > \mu_{\text{crit}} \hspace{1cm} (5.15. b)$$

$$\delta_u = \frac{5L^2 \phi_u}{72} + \frac{M_u L^2}{27M_{bcr}} \phi_{bcr}; \hspace{1cm} \mu < \mu_{\text{crit}} \hspace{1cm} (5.15. c)$$
Eqs. (5.14) and (5.15) are also applicable for calculating the midspan deflection at any given moment. For a beam subjected to elastic moment $M_e$ below the bilinear cracking moment $M_{bcr}$, the elastic deflection is calculated by replacing $\varphi_{bcr}$ with elastic curvature $\varphi_e$ corresponding to $M_e$ in Eq. (5.14a) or (5.15a) depending on the bending test setup. Likewise, if a beam is subjected to the postcrack moment $M_{pc}$, the deflection-hardening or softening is calculated by replacing $M_u$ and $\varphi_u$ with $M_{pc}$ and $\varphi_{pc}$ in either Eq. (5.14b) and (5.14c) for the three-point or Eq. (5.15b) and (5.15c) for the four-point bending, respectively. It is noted that the moment and curvature distributions of a cantilever beam subjected to point load at the tip are similar to the three-point bending half model in Fig. 5.6. Therefore, Eqs. (5.14a)–(5.14c) are also applicable to calculate the tip deflection of a cantilever beam.

5.2.6. Algorithm to Predict Load-Deflection Responses

Load-deflection response of the three- and four-point bending beam can be calculated by the following procedure.

1) For a given set of material parameters $E$, $\varepsilon_{cr}$, $\mu$, $\omega$, $\beta_{tu}$, $\lambda_{cu}$, and beam section $b$ and $h$, calculate normalized ultimate top compressive strain $\lambda_u$, as the smaller of $\lambda_{cu}$ and $\lambda_{tu}$ determined from Eqs. (5.5) and (5.7). The cracking moment and curvature $M_{cr}$, $\varphi_{cr}$ are calculated from Eqs. (5.3) and (5.4), respectively. The next step is to calculate the normalized ultimate moment $M'_{u}$ by the expression given in Table 5.3. For an FRC with $\mu < \mu_{crit}$ and tested in the three-point bending, the additional plastic zone $L_p$ is needed to describe the length of the localization in the postpeak range. This length $L_p$
is not needed in material with $\mu > \mu_{crit}$ or in the four-point bending that the entire midsection is assumed undergoes crack localization.

2) Use the regression Eq. (5.13) to calculate the intersection point of two straight lines in the bilinear moment curvature model ($\phi'_{it}$, $M'_{it}$). For deflection-hardening FRC ($\mu > \mu_{crit}$), $\phi'_{it}$ and $M'_{it}$ become a bilinear cracking moment curvature $\phi'_{bcr}$ and $M'_{bcr}$, respectively. Three controlling points for the bilinear model are

$$\phi = \begin{bmatrix} 0 \\ \phi'_{bcr} \\ \phi'_u \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ M'_{bcr} \\ M'_u \end{bmatrix}$$

For deflection-softening FRC ($\mu < \mu_{crit}$), the intersection point ($\phi'_{it}$, $M'_{it}$) obtained from Eq. (5.13) is less than the original normalized cracking point ($\phi'_{cr}$, $M'_{cr}$), which is equal to the bilinear cracking moment curvature $\phi'_{bcr}$ and $M'_{bcr}$. The normalized reduced cracking moment $M'_{cr2}$ at $\phi'_{bcr}$ is need to be interpolated by Eq. (5.12) to yield four controlling points for the bilinear model in this case

$$\phi = \begin{bmatrix} 0 \\ \phi'_{bcr} \\ \phi'_{bcr} \\ \phi'_u \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ M'_{bcr} \\ M'_{cr2} \\ M'_u \end{bmatrix}$$

3) Substitute the individual element of vectors $\phi$ and $M$ from either Eq. (5.16) or (5.17) in Eq. (5.14) for the midspan deflection of the three-point bending or in Eq. (5.15) for the four-point bending. If needed, the intermediate values of $\phi$ and $M$ between the controlling points defined by Eq. (5.16) or (5.17) can be interpolated and substituted.
in either Eq. (5.14) or (5.15) to get more response data between those controlling points.

4) From the approximate bilinear moment curvature diagram, the total load \( P_i \) at a given point \( i \) along the diagram can be calculated by:

\[
P_i = \frac{2M_i}{X} \quad \text{for } \phi_i = 0...\phi_u
\]  
(5.18)

where \( X = L/2 \) for three-point bending and \( X = L/3 \) for four point bending.

5) Repeat steps 3 and 4 to obtain an approximate load-deflection curve.

5.2.7. Parametric Studies of Material Parameters

Parametric studies were used to demonstrate the sensitivity of the material behaviors relative to the input variables. The material model for FRC as shown in Fig. 5.2 is prescribed by six parameters: \( E, \varepsilon_{cr}, \mu, \omega, \beta_{tu} \) and \( \lambda_{cu} \). As shown in Table 5.3, the characteristics of normalized moment-curvature response are controlled by two material parameters: Normalized postpeak tensile strength \( \mu \) and normalized compressive yield strain \( \omega \). As mentioned earlier in the description of the material models, \( \omega \) is also known as compressive yield strength ratio when compressive modulus is the same as the tensile modulus. Fig. 5.7 shows the effect of \( \mu \) on the neutral axis depth ratio \( k \) and the normalized moment-curvature response. Fig. 5.7(a) shows that for small values of postpeak tensile strength, \( \mu \), the reduction in the value of \( k \) and the normalized moment-curvature response is quite rapid, and the response terminates at a relatively low top compressive strain \( \lambda \). More ductile systems (high \( \mu \)) result in a gradual reduction of \( k \) values as the input strain is increased, allowing large deformations up to failure.
Fig. 5.7(b) reveals that parameter $\mu$ directly affects the moment-curvature response. Very brittle materials ($\mu \leq 0.01$) by default have a moment capacity equal to the normalized moment at cracking $M'_{cr} = 1$ with an insignificant postpeak capacity. Very ductile materials ($\mu = 1.0$) show the continuing increase of the moment capacity beyond the normalized cracking moment $M'_{cr} = 1$ to levels as high as 2.6. Noted that when a value of $\mu$ is 0.33 in Fig. 5.7(b), the normalized ultimate moment $M'_u$ is 0.95, that is slightly less than $M'_{cr} = 1.00$. In order to obtain elastic perfectly moment curvature response for material having $\omega = 10$, $\mu$ must be at least $\mu_{crt} = 0.345$ as calculated by Eq. (5.11).

Fig. 5.8 shows the effect of the compressive yield strength ratio, $\omega$, to the neutral axis depth ratio, $k$, and to the normalized moment-curvature response. As shown in Fig. 5.8(a), the changes in $k$ value is more pronounced for $\omega$ in the low range (1–4) and marginally affected in the higher ranges (7–10). Similarly, the normalized moment-curvature diagram is sensitive to the change of $\omega$ in the low range (1–4), but not at higher ranges (7–10). This indicates that the effect of the compressive strength of typical FRC on the flexural behavior is marginal especially when the strength ratio is relatively high. It also implies that the yield compressive stress $\sigma_{cy}$ defined in Fig. 5.2(b) can be reasonably assumed in the 0.6–0.9 of the ultimate compressive strength $f'_{c}$, without significantly affecting the predicted load-deflection response.

5.2.8. Simulations of Strain-Softening Steel Fiber Reinforced Concrete

Four sets of SFRC specimens containing hook-end fibers at three volume fraction levels of 0.5, 1.0, and 1.5% and tested under three- and four-point bending were selected to demonstrate the algorithm for the load-deflection response (Lim et al. 1987a,b).
details of these four series and their parameters used in the simulations are listed in Table 5.4, and the specimens were designated as H22, H3, H1 and H21.

Figs. 5.9(a and b) show the tension model and the predicted flexural response under the three-point bending for Specimens H22 with \( V_f = 1.0\% \). The solid line in Fig. 5.9(a) was obtained by fitting the tension model to the experimental tension test data, and parameters for the model are listed in the first line of Table 5.4. The solid curve in Fig. 5.9(b) represents the predicted flexural response obtained by the simulation process. It clearly reveals that the direct use of the tension data underpredicts the flexural response. By examining the load-deflection response as shown in Fig. 5.9(b) and Table 5.4, the nominal flexural stress at the peak is 6.3 MPa, which is computed from the load at cracking \( P_{cr} = 14,000 \text{ N} \), clear span \( L = 300 \text{ mm} \), beam width \( b = 100 \text{ mm} \), and beam depth \( h = 100 \text{ mm} \). When compared to the uniaxial tensile strength \( \varepsilon_{cr}E \) of 2.95 MPa in Table 5.4, the ratio of flexural stress to uniaxial stress is 2.13. By scaling the tension model by this ratio, the predicted response will match the experimental result. A uniform increase in the tension capacity can be achieved by increasing the cracking strain \( \varepsilon_{cr} \), since other strains and related stress measures defined in Fig. 5.2 will be amplified by the same factor. In this case, however, it is observed that a uniform increase in tension capacity by raising \( \varepsilon_{cr} \) by a factor of 2.13 and decreasing \( \mu \) to 0.30 results in a good prediction of the response as shown by the dash line in Fig. 5.9(b). Figs. 5.9(c and d) show the use of tension data to predict the flexural response of Mixture H3 with \( V_f = 1.5\% \). The overall prediction is reasonable with the simulation exhibiting a slightly stiffer response than the experimental data. Note, that no modification to the tension
model is necessary for this case, since the material is quite ductile, therefore, the brittleness effects are diminished.

Fig. 5.10 shows the direct use of tensile response to predict the flexural response of the four-point bending test. Fig. 5.10(a) shows the uniaxial tension test result of Mix H1 ($V_f = 0.5\%$), the fitted tension model, and modified tension model that yields the prediction and the best fit to flexural test data. Fig. 5.10(b) confirms that the fitted model underestimates the experimental load-deflection curve. At cracking load $P_{cr} = 6,000$ N, clear span $L = 750$ mm, beam width $b = 100$ mm, and beam depth $h = 100$ mm, the nominal flexural stress for the four-point bending is 4.50 MPa, which is 1.61 times the uniaxial tensile strength. For this Mix H1 with a low fiber content $\mu = 0.24 < \mu_{crit} = 0.35$, the modification of the tension model by raising $\varepsilon_{cr}$ with a factor of 1.61 results in a reasonable prediction as the dashed line shown in Fig. 5.10(b). For Mix H21 ($V_f = 1.0\%$) shown in Figs. 5.10(c and d), the fitted tension model slightly underpredicted the experimental result and with a modifying factor of 1.09 to $\varepsilon_{cr}$ for the increasing tensile capacity, it leads to the best fit of the predicted response.

5.2.9. Discussions

Three observations are noted from the simulations presented in the previous section. First, there is a discrepancy between the uniaxial tension and the nominal flexural strength results such that using uniaxial test data underpredicts the flexural strength in beam test. This is related to the stress gradient that exists in the flexural test, both along the span length and depth of the beam. Simply stated, the uniform stress in the uniaxial test has a higher probability of localizing at a defect in material in order to initiate the
crack than the triangular stress in the bending test. Second, the variation of the modified parameters presented in Table 5.4 indicates that the three-point bending varies to a wider degree than the four-point bending test. This verifies the accepted knowledge that the four-point bending test is a better representative of the tension test than the three-point bending test, due to the uniformity in stress distribution. Finally, the geometrical effect seems to decrease in both the three- and four-point bending when normalized postpeak tensile strength increases. Higher postpeak strength implies there are more fibers in the material to suppress the cracks initiation at critical defects such that the uniaxial test and flexural test become less sensitive to the probability distribution of the defects.

5.3. Extended Model for Strain-Softening and Strain-Hardening

The idealized material model for strain-softening FRC presented in Fig. 5.2 was extended by adding more parameters in order to describe wider ranges of material responses. Using more parameters in the derivations will result in more complicate closed form solutions for moment curvature response. Nevertheless, the flexural behaviors of both classes of FRC (strain-softening and hardening) can be predicted by only one model.

5.3.1. Material Models

Fig. 5.11 presents a uniaxial parameterized model for strain-softening and hardening fiber reinforced concrete with two intrinsic material parameters: elastic tensile modulus, $E$ and cracking tensile strain, $\varepsilon_{cr}$. Three normalized parameters ($\eta$, $\mu$ and $\beta_{tu}$) are used to describe tension model and the other three ($\omega$, $\gamma$ and $\lambda_{cu}$) are for compression model.

Fig. 5.11(a) shows the tension model that is described by a trilinear with an elastic range defined by tensile modulus $E$. The linear response is terminated at the cracking
strain $\varepsilon_{cr}$ and cracking stress $\sigma_{cr} = \varepsilon_{cr}E$. Then the response after cracking is soften defined by postcrack tensile modulus $E_{cr} = \eta E$. By setting $\eta$ to either a negative or a positive value, the same model can be used to simulate strain-softening or strain-hardening materials. The third region is defined by a postpeak tensile strength $\sigma_p = \mu\varepsilon_{cr}E$, which begins at the transition strain $\varepsilon_{trn} = \alpha\varepsilon_{cr}$. This transition strain can be a point at softening stress (solid line) in strain-softening material or a point at peak stress (dash line) in strain-hardening material. The normalized postpeak stress $\mu$ can be set to the value less than one for strain-softening and any value for strain-hardening. This tension model allows stress to be continuous or discontinuous at transition strain due to arbitrary value of $\mu$. Finally, the tensile response terminates at the ultimate tensile strain level of $\varepsilon_{tu} = \beta_{tu}\varepsilon_{cr}$.

Fig. 5.11(b) shows the compression model, in which the compressive modulus $E_c = \gamma E$ can be different from the tension by a factor of $\gamma$. The linear elastic portion ends at a compressive yield strain $\varepsilon_{cy} = \omega\varepsilon_{cr}$ and compressive yield stress $\sigma_{cy} = \gamma\omega\varepsilon_{cr}E$. After yielding the stress remains constant and terminates at the ultimate compressive strain $\varepsilon_{cu} = \lambda_{cu}\varepsilon_{cr}$. The stress–strain relationship for tension and compression can be expressed as

\[
\sigma_t(\varepsilon_t) = \begin{cases} 
E\varepsilon_t & 0 \leq \varepsilon_t \leq \varepsilon_{cr} \\
E\varepsilon_{cr} + E_{cr}(\varepsilon_t - \varepsilon_{cr}) & \varepsilon_{cr} \leq \varepsilon_t \leq \varepsilon_{trn} \\
\mu E\varepsilon_{cr} & \varepsilon_{trn} \leq \varepsilon_t \leq \varepsilon_{tu} \\
0 & \varepsilon_t > \varepsilon_{tu}
\end{cases}
\]  
\[\text{(5.19)}\]

\[
\sigma_c(\varepsilon_c) = \begin{cases} 
E_c\varepsilon_c & 0 \leq \varepsilon_c \leq \varepsilon_{cy} \\
E_c\varepsilon_{cy} & \varepsilon_{cy} \leq \varepsilon_c \leq \varepsilon_{cu} \\
0 & \varepsilon_c > \varepsilon_{cu}
\end{cases}
\]  
\[\text{(5.20)}\]
where $\sigma_t$, $\sigma_c$, $\varepsilon_t$, and $\varepsilon_c$, are tensile and compressive stresses and strains, respectively. In order to derive the closed form solutions for moment–curvature response in non-dimensional forms, all modulus, strains and stresses used in the models are normalized with respect to $E$, $\varepsilon_{cr}$ and $E\varepsilon_{cr}$, respectively.

\[
\alpha = \frac{\varepsilon_{tn}}{\varepsilon_{cr}}; \quad \beta_{tu} = \frac{\varepsilon_{tu}}{\varepsilon_{cr}}; \quad \omega = \frac{\varepsilon_{cy}}{\varepsilon_{cr}}; \quad \lambda_{cu} = \frac{\varepsilon_{cu}}{\varepsilon_{cr}}
\]

(5.21)

\[
\eta = \frac{E_{cr}}{E}; \quad \gamma = \frac{E_c}{E}
\]

(5.22)

\[
\mu = \frac{\sigma_p}{E\varepsilon_{cr}}
\]

(5.23)

The normalized tensile strain at the bottom fiber $\beta$ and compressive strain at the top fiber $\lambda$ are defined as:

\[
\beta = \frac{\varepsilon_{bot}}{\varepsilon_{cr}}; \quad \lambda = \frac{\varepsilon_{top}}{\varepsilon_{cr}}
\]

(5.24)

They are linearly related through the normalized neutral axis parameter, $k$.

\[
\frac{\lambda\varepsilon_{cr}}{kd} = \frac{\beta\varepsilon_{cr}}{d-kd} \quad \text{or} \quad \lambda = \frac{k}{1-k}\beta
\]

(5.25)

Substitution of all normalized parameters defined in Eqs. (5.21) - (5.24) into Eqs. (5.19) and (5.20) results in the following normalized stress strain models at bottom and top fibers:

\[
\frac{\sigma_t(\beta)}{E\varepsilon_{cr}} = \begin{cases} 
\beta & 0 \leq \beta \leq 1 \\
1+\eta(\beta-1) & 1 < \beta \leq \alpha \\
\mu & \alpha < \beta \leq \beta_{tu} \\
0 & \beta_{tu} < \beta
\end{cases}
\]

(5.26)
\[
\frac{\sigma_e(\lambda)}{E\varepsilon_{cr}} = \begin{cases} 
\gamma\lambda & 0 \leq \lambda \leq \omega \\
\gamma\omega & \omega < \lambda \leq \lambda_{cu} \\
0 & \lambda_{cu} < \lambda
\end{cases} \tag{5.27}
\]

5.3.2. Derivation of Moment-Curvature Response

In the derivation of moment-curvature diagram for a rectangular cross section with a width \(b\) and depth \(h\), the Kirchhoff hypothesis of plane section remaining plane for flexural loading is applied. Unlike the strain-softening model that uses normalized compressive strain at top fiber \((\lambda = \varepsilon_{\text{top}}/\varepsilon_{cr})\) as an independent variable to impose flexural deformation, the extended model uses normalized tensile strain at bottom fiber \((\beta = \beta_{\text{bot}}/\varepsilon_{cr})\) instead. This is due to the tension model is divided in three stages while the compression model is only two stages. Using the \(\lambda\) as an independent variable requires solving the value of \(\lambda\) at transition of \(\beta\) from stage 1 to 2 and stage 2 to 3, resulting in cumbersome procedure.

By assuming linear strain distribution across the depth and ignoring shear deformation, the stress–strain relationships in Fig. 5.11(a and b) are used to obtain the stress distribution across the cross section as shown in Fig. 5.12 at three stages of imposed tensile strain: \(0 < \beta \leq 1, 1 < \beta \leq \alpha, \alpha < \beta \leq \beta_{tu}\). For stage 2 and 3 there are two possible scenarios: the compressive strain at top fiber is either elastic \((0 < \lambda \leq \omega)\) or plastic \((\omega < \lambda \leq \lambda_{cu})\). These cases will be treated in subsequent sections.

Normalized heights of compression and tension zones with respect to beam depth \(h\) and the normalized magnitudes of stress at the vertices with respect to the cracking stress \(E\varepsilon_{cr}\) are presented in Tables 5.5 and 5.6, respectively. The area and centroid of stress in each
zone represent the force components and lines of action. Their normalized values with respect to cracking tensile force $bhE\varepsilon_{cr}$ and beam depth $h$ are presented in Tables 5.7 and 5.8, respectively.

Table 5.9 shows the steps in determination of net section force, moment, and curvature at each stage of applied tensile strain, $\beta$. The net force is obtained as the difference between the tension and compression forces, equated to zero for internal equilibrium, and solved for the neutral axis depth ratio $k$. The expressions for net force in stage 2 and 3 are in the quadratic forms and result in two solutions for $k$. With a large scale of numerical tests covering a practical range of material parameters, only one solution of $k$ yields the valid value in the range $0 < k < 1$ and it is presented in Table 5.10. The internal moment is obtained by operating on the force components and their distance from the neutral axis. The curvature is determined as the ratio of compressive strain at top fiber ($\varepsilon_{ctop} = \lambda\varepsilon_{cr}$) to the depth of neutral axis $kh$. The moment $M$ and curvature $\phi$ at each stage are then normalized with respect to the values at cracking $M_{cr}$ and $\phi_{cr}$, respectively and their closed form solutions are presented in Table 5.10.

$$M = M' M_{cr}; \quad M_{cr} = \frac{1}{6} bh^2 E\varepsilon_{cr}$$ (5.28)

$$\phi = \phi' \phi_{cr}; \quad \phi_{cr} = \frac{2\varepsilon_{cr}}{h}$$ (5.29)

As mentioned earlier, the compressive strain at the top fiber $\lambda$ in stage 2 or 3 could be either in elastic ($0 < \lambda \leq \omega$) or plastic ($\omega < \lambda \leq \lambda_{cu}$) range, depending on the applied tensile strain $\beta$ and neutral axis parameter $k$. The range can be identified by assuming $\lambda < \omega$ [Fig. 5.12(b.1) or (c.1)] and using the expression $k_{21}$ or $k_{31}$ in Table 5.10 to
determine $\lambda$ from Eq. (5.25). If $\lambda < \omega$ holds true, the assumption is correct, otherwise $\lambda > \omega$ and the expression $k_{22}$ or $k_{32}$ is used instead. Once, the neutral axis parameter $k$ and the applicable case are determined, the appropriate expressions for moment and curvature in Table 5.10 and Eqs. (5.28) and (5.29) are used.

5.3.3. Crack Localization Rules

When a flexural specimen is loaded beyond the peak strength, the load decreases and two distinct zones develop as the deformation localizes in the cracking region while the remainder of the specimen undergoes general unloading. To correlate the stress-crack width relationship into the stress–strain approach, localization of major cracks is simulated as an average response over the crack spacing region. Results are used as a smeared crack in conjunction with the moment–curvature diagram to obtain load deformation behavior.

Fig. 5.13(a) presents the schematic moment–curvature diagram with crack localization rules and Fig. 5.13(b) shows a four point bending test with localization of smeared crack occurs in the mid-zone; while the zones outside the cracking region undergo unloading during softening (Ulfkjaer et al. 1995; Olesen 2001). The length of the localized zone is defined as “$cX$” representing product of a normalized parameter $c$ and loading point spacing $X = L/3$, where $L$ is the clear span. For the simulations of fiber reinforced composites in this paper, it was assumed that cracks were uniformly distributed throughout the mid-zone and a value of $c = 0.5$ was used.

Moment distribution along the length of a beam is obtained by static equilibrium and the corresponding curvature is obtained from a moment–curvature relationship. As shown
by a solid curve in Fig. 5.13(a), a typical moment–curvature diagram is divided into two portions: an ascending curve from 0 to $M_{\text{max}}$ and a descending curve from $M_{\text{max}}$ to $M_{\text{fail}}$. For a special case of very low-fiber volume fraction where an ascending curve from 0 to $M_{\text{max}1}$, representing the tensile cracking strength, is followed by a sharp drop in the postpeak response, the postpeak moment–curvature response exhibits two portions: a descending curve from $M_{\text{max}1}$ to $M_{\text{low}}$ and ascending again from $M_{\text{low}}$ to $M_{\text{max}2}$. In this case, there are two local maxima, which either point could be the global maximum.

To predict load–deflection response, an array of load steps is derived from a series of discrete data points along a moment–curvature diagram. For each load step, the moment and corresponding curvature distribution along the beam are calculated. While the specimen is loaded from 0 to $M_{\text{max}}$ (or $M_{\text{max}1}$), the ascending portion of the diagram is used. Beyond the maximum load, as the specimen undergoes softening, the curvature distribution depends on the localized or non-localized zones and its prior strain history (uncracked or cracked). For an uncracked section, the curvature unloads elastically. If the section has been loaded beyond $M_{\text{cr}}$, the unloading curvature of cracked sections follows a quasi-linear recovery path expressed as:

\[
\phi_j = \phi_{j-1} - \xi \frac{(M_{j-1} - M_j)}{K}
\]

(5.30)

where $\phi_{j-1}$ and $M_{j-1}$ represent the previous moment–curvature state and $\phi_j$ and $M_j$ are the current state. $K = M_{cr}/\phi_{cr}$ represents the flexural stiffness of uncracked section. The unloading factor $\xi$ is between 0 and 1; $\xi = 0$ indicates no curvature recovery while $\xi = 1$ is unloading elastically with initial stiffness $K$. An unloading factor $\xi = 0$ was used in the
present study under the assumption that cracks do not close when material softens in displacement control. For a section in the localized zone, the unloading curvature is determined from the descending portion of the moment–curvature diagram ($M_{\text{max}}$ to $M_{\text{fail}}$) or ($M_{\text{max}1}$ to $M_{\text{low}}$). For a special case of very low-fiber content that the moment–curvature diagram is divided into three portions, the curvature corresponding to the load step beyond the $M_{\text{low}}$ is determined by the third portion ($M_{\text{low}}$ to $M_{\text{max}2}$).

5.3.4. Algorithm to Predict Load-Deflection Responses

The load–deflection response of a beam can be obtained by using the moment–curvature response, crack localization rules, and moment-area method as follows.

1) For a given cross section and material properties, the normalized tensile strain at the bottom fiber $\beta$ is incrementally imposed to generate the moment–curvature response using Eqs. (5.28) and (5.29), and the expressions given in Table 5.10. For each value of $\beta$ in stage 2 and 3, the condition for compressive stress $0 < \lambda < \omega$ or $\omega < \lambda \leq \lambda_{\text{cu}}$ is verified in advance of moment–curvature calculation.

2) Since a moment–curvature diagram determines the maximum load allowed on a beam section, the discrete moments along the diagram are used to calculate the applied load vector $P = 2M/X$ as shown in Fig. 5.13(b).

3) The beam is segmented into finite sections. For a given load step, use static equilibrium to calculate moment distribution along the beam and use moment–curvature relationship with crack localization rules to identify the curvature.
4) The deflection at mid-span is calculated by numerical moment-area method of discrete curvature between the support and mid-span. This procedure is applied at each load step to until a complete load–deflection response is obtained.

5.3.5. Parametric Studies of Material Parameters

Two sets of parametric studies were conducted to address the behavior of strain-softening and strain-hardening materials. The flexural strength and ductility for each material parameter studied were expressed as the normalized moment–curvature response, which is independent of section size and cracking tensile strength.

Fig. 5.14 presents the parametric study of a typical strain-softening FRC with material parameters specified as a compressive yield strength ratio $\gamma = 10$, normalized ultimate compressive strain $\lambda_{cu} = 30$ and normalized ultimate tensile strain $\beta_{tu} = 150$. For each case of study, all parameters were held constant to the typical values while the parameters subjected to study were varied. In order to avoid a discontinuous tensile response at the normalized transition strain $\alpha$ for strain-softening materials, the normalized postcrack tensile modulus $\eta$ is determined by $\alpha$ and normalized postpeak stress $\mu$

$$\eta = -\frac{(1 - \mu)}{(\alpha - 1)}$$  \hspace{1cm} (5.31)

Fig. 5.14(a) depicts the compression and tension model with the transition strain $\alpha$ varied from 1.01 to 15. Fig. 5.14(b) reveals that an increase in the transition strain $\alpha$, increases both flexural strength and ductility. Fig. 5.14(c) depicts the material model with the postpeak strength $\mu$ varied from 0.01 to 0.99, simulating a range of brittle concrete to elastic–perfectly plastic tensile response of high volume fraction fiber reinforced
concrete. Fig. 5.14(d) shows that moment–curvature diagram is quite sensitive to the variations in parameter \( \mu \) as it significantly affects the peak and postpeak response. The flexural response changes from a brittle to ductile material as \( \mu \) changes from 0.01 to 0.99. In order to study the effect of compressive stiffness, the range of normalized compressive modulus \( \gamma \) and compressive yield strain \( \omega \) were used together to represent the increase in relative compressive to tensile stiffness from \( 0.1 < \gamma/\omega < 16 \) (1/10 to 4/2.5) at a fixed compressive yield strength ratio \( \gamma\omega = 10 \) as shown in Fig. 5.14(e). Fig. 5.14(f) reveals that the changes in the relative stiffness slightly affect the peak moment from 1.7 to 2.1 and marginally increases the stiffness of the moment–curvature response. It is also concluded that the \( \gamma \) and \( \omega \) have a marginal effect on the predicted moment–curvature response as long as the compressive strength is about one order of magnitude higher than the cracking tensile strength. This conclusion is the same as the parametric study shown in Fig. 5.8(b) that uses the strain-softening model developed in section 5.2.

Fig. 5.15 presents the parametric study of a typical strain-hardening material, with a compressive yield strength ratio \( \gamma\omega = 10 \), normalized ultimate compressive strain \( \lambda_{cu} = 30 \) and normalized ultimate tensile strain \( \beta_{tu} = 150 \) similar to the previous case study. The postpeak tensile response was ignored for this case by setting \( \mu \) to a very low value of 0.01. Fig. 5.15(a) shows the compression and tension model of a typical strain-hardening material with varying \( \alpha = 1.01–15 \). Fig. 5.15(b) shows that the increase in \( \alpha \) directly increases the normalized moment and curvature. Figs. 5.15(c and d) demonstrate that increasing the postcrack tensile modulus \( \eta \) also significantly increases the
moment–curvature diagram. Similar to the strain-softening materials, the increase in relative compressive to tensile stiffness at constant compressive yield strength ratio $\gamma_\omega = 10$ [as shown in Fig. 5.15(e)] has a subtle effect to the moment–curvature response as shown in Fig. 5.15(f). These results indicate that the most significant parameters affecting the moment capacity are the normalized transition strain $\alpha$ and postcrack tensile modulus $\eta$.

5.3.6. Simulations of Strain-Softening and Hardening Fiber Reinforced Concrete

The algorithm to simulate load–deflection response of a beam under four point bending test was used as a predictive tool to study three classes of materials: SFRC with 0.5% and 1.0% volume fraction representing strain-softening deflection-softening and strain-softening deflection-hardening material, respectively, and ECC with 2.0% volume fraction representing strain-hardening material.

5.3.6.1. Simulation of Steel Fiber Reinforced Concrete (SFRC)

Two mixes of SFRC (H1 and H21) that used hook-end fibers at volume fraction levels of 0.5% and 1.0% were selected from the literature (Lim and Paramasivam 1987a,b) to demonstrate the algorithm to predict load–deflection responses. Tensile “dog bone” specimens of 100 x 70 x 200 mm in net dimensions with an enlarged width of 100 mm at the ends were used. The flexural four point bending specimens were 100 x 100 x 1000 mm with a clear span of 750 mm. The average material properties were: compressive strength $f_{c'} = 34$ MPa, initial compressive modulus $E_{ci} = 28.5$ GPa and initial tensile modulus $E = 25.4$ GPa. The cracking tensile strain $\varepsilon_{cr}$ for mix H1 and H21 were 110.6 and 116.2 microstrains, respectively.
As shown in the parametric studies, the material parameters for compression model have a marginal effect to the predicted flexural response as long as the compressive strength is about one order of magnitude higher than the cracking tensile strength. Therefore, typical values for compression parameters can be estimated without severely affecting the results. The compressive yield stress $\sigma_{cy}$ was assumed to be $0.85f_{c'}$ and compressive modulus $E_c$ was estimated to be $0.85E_{ci}$. The normalized compressive modulus $\gamma$ was then obtained by $E_c/E$ and the normalized compressive yield strain by $\omega = \sigma_{cy}/(E_c\epsilon_{cr})$. The range of ultimate compressive strain $\epsilon_{cu}$ between 0.0035 and 0.004 was suggested by several researchers (Swamy and Al-Ta’an 1981; Hassoun and Sahebjam 1985). The value of 0.004 was selected in this study and the corresponding normalized value was calculated by $\lambda_{cu} = \epsilon_{cu}/\epsilon_{cr}$. The material parameters for tension model were determined by fitting the model to the uniaxial tension test result as shown by the solid line in Figs. 5.16(a and c). All other parameters used in the simulation of flexural bending of mix H1 and H21 are provided in the figures.

For deflection-softening material represented by mix H1, the normalized parameters used in simulation were: the postpeak tensile strength $\mu = 0.24$, postcrack tensile modulus $\eta = -0.76$, transition strain $\alpha = 2.0$, ultimate tensile strain $\beta_{tu} = 136$, compressive modulus $\gamma = 0.95$ and compression yield strain $\omega = 10.8$. The solid curve in Fig. 5.16(b) shows the predicted flexural response of deflection-softening material (mix H1) from the simulation process. It is shown that use of uniaxial tension data underpredicts the flexural response for this class of material. This is attributed to differences in the stress distribution profiles of the two test methods as previously discussed in section 5.2.9. By using a single scaling
parameter that is applied to the cracking tensile strain $\varepsilon_{cr}$, the associated strains and stresses increase by the same factor, resulting in a uniform increase in material strength. An inverse analysis by trial and error was conducted to identify the appropriate scaling parameter for this case and it found that the material model should be scaled up uniformly by 40%. The dash lines in Figs. 5.16(a and b) show the modified strength and the new prediction, which provides a better match to the experimental results. If tensile and flexural responses of several samples are utilized, the inverse analysis procedure can be used to establish the statistical relationship between the tensile and flexural responses as have been done for other brittle materials (Noguchi et al. 1990).

For deflection-hardening materials represented by mix H21, the solid curve of tension model that fits to the uniaxial tension test data is shown in Fig. 5.16(c). The normalized parameters used in simulation were: the postpeak tensile strength $\mu = 0.8$, postcrack tensile modulus $\eta = -0.1$, transition strain $\alpha = 3.0$, ultimate tensile strain $\beta_{tu} = 129$, compressive modulus $\gamma = 0.95$ and compression yield strain $\omega = 10.3$. The predicted flexural response is shown as a solid line in Fig. 5.16(d). The algorithm slightly underestimates the flexural response. The inverse analysis found that the strength of the uniaxial models should be increased by 8% for a reasonable prediction of flexural results as shown by the dashed lines in Figs. 5.16(c and d). Note that the correlation of experimental and simulated responses in the deflection-hardening range is quite reasonable; the discrepancy between the fitted and modified tension models is much lower than that of deflection-softening materials. This indicates that the model prediction for the deflection-hardening materials is quite reasonable.
5.3.6.2. Simulation of Engineered Cementitious Composite (ECC)

An ECC mix that used polyethylene fibers at volume fraction levels of 2.0% from the literature (Maalej and Li 1994; Maalej 2004) was selected to demonstrate the ability of the algorithm to predict load–deflection response for strain-hardening materials. The flexural specimens for the four point bending test were 76.2 x 101.6 x 355.6 mm with a clear span of 304.8 mm. The average material properties were: compressive strength $f_c' = 60$ MPa. The initial compressive modulus $E_{ci} = 4.35$ GPa was obtained by back-calculation of the initial flexural load–deflection response. The initial tensile modulus $E = 4.75$ GPa and the cracking tensile strain $\varepsilon_{cr} = 600$ microstrains were obtained directly from the uniaxial tensile test results.

The compressive yield stress $\sigma_{cy}$ was assumed to be $0.80f_c'$ and compressive modulus $E_c$ was estimated equal to $E_{ci}$. The ultimate compressive strain $\varepsilon_{cu}$ was assumed to be 0.012. The material parameters for tension model were determined by fitting the model to the uniaxial tension test result as shown by the solid line in Fig. 5.17(a). All normalized parameters used in the simulation are provided in the same figure: the postpeak tensile strength $\mu = 1.49$, postcrack tensile modulus $\eta = 0.009$, transition strain $\alpha = 95.0$, ultimate tensile strain level $\beta_{tu} = 117$, compressive modulus $\gamma = 0.92$ and compressive yield strain $\omega = 18.4$. The solid curve in Fig. 5.17(b) shows the predicted flexural response obtained by the simulation process. The prediction for the strain-hardening material during the pre- and post-crack stages agreed well with the experimental results, which implies the formation of the distributed crack system in ECC can be adequately described by the smeared pseudo-strain in the material model.
5.3.7. Discussions

The three simulations of fiber reinforced concrete materials, ranging from low to high fiber contents that material response changing from strain-softening to hardening, indicate that as the postpeak tensile strength of material increases, the use of uniaxial response to predict flexural response becomes more accurate. It also implies that the increase of fiber contents in the specimens can effectively suppress the flaws from initiating crack that leads to premature failure. Therefore, the flaw size distribution in material is less sensitive to the constant vs. linear distribution of tensile stress patterns in uniaxial and flexural specimens, respectively.

5.4. Chapter Conclusions

Uniformly fiber reinforced concrete composites can be viewed as a homogenized material such that the flexural behaviors can be predicted by using uniaxial material models. This chapter presents two parameterized material models; the first one is applicable for strain-softening FRC only while the second one is applicable for both strain-softening and strain-hardening FRC. The obtained closed form solution for the first model is simpler but less accurate in modeling flexural response than the second model. However, the second model results in a more complex set of equations than the first one. Engineers need to balance between the simplicity and the accuracy when choosing the set of closed form solutions for their work.

In the strain-softening FRC model, the closed form solution for moment-curvature relationship was derived and then simplified to two bilinear models for its two subclasses: deflection-softening and deflection-hardening. The critical normalized
postpeak tensile strength that defines these two subclasses was derived explicitly. For typical FRC that compressive yield strength varies from 6 to 12 times the cracking tensile strength, the critical normalized postpeak tensile strength is 0.345. The parametric study of normalized postpeak tensile strength parameter confirms that this number changes the flexural response from deflection-softening to hardening. Finally, the two bilinear moment-curvature models were then used to derive closed form solutions for the midspan deflections of three- or four-point bending by moment-area method.

The strain-softening FRC model is extended by adding three more parameters in order to capture other material characteristics: parameter for postcrack tensile modulus that can be set to a negative or positive value for modeling strain-softening or strain-hardening material; parameter for transition strain can be used to define strain at the end of softening postcrack response or strain at the ultimate tensile strength; and parameter that allows compressive modulus be different from the tensile modulus. The close form solutions for moment-curvature relationship was derived and obtained a more complex set of equations than the strain-softening FRC model due to more parameters included in the derivation. The equations were then used in parametric studies for strain-softening and strain-hardening materials. For a typical strain-softening material, the most important factors to flexural strength and ductility are the postpeak tensile strength and the strain at the end of softening postcrack response. For a typical strain-hardening material, both tensile strain at ultimate tensile strength and postcrack modulus are almost equally important to the flexural behavior of a beam. Strain-softening and hardening materials share the similarity that the change of relative compressive to tensile stiffness slightly
affects the flexural response as long as the compressive strength is several times higher than the tensile strength.

Both strain-softening model and the extended model for strain-softening/hardening were used to predict the load-deflection response of different fiber reinforced concrete systems. The simulations revealed that the use of uniaxial tensile data tends to underpredict the flexural response for the material that has a relatively low postpeak tensile strength. This is in part due to a difference in stress distribution between the uniaxial tension and bending tests. The underpredicted load deformation response can be corrected by increasing strength of the uniaxial material model using a scaling parameter to the cracking strain. With this approach, other associated strains and stresses will subsequently be increased by the same factor. With proper scaling parameters, the predicted responses agreed well with experimental observations. It is also observed that as fiber content, or bond is increased such that the postpeak tensile strength is improved, the size effect observed in predicting the response of flexural samples tends to be reduced and the use of uniaxial responses to predict flexural response becomes more accurate.

**Notation**

*The following symbols are used in this chapter:*

\[ b \]  = beam width;
\[ C \]  = a parameter for normalized moment in Table 5.10;
\[ c \]  = a parameter to define localized crack length;
\[ D \]  = a parameter for neutral axis depth ratio in Table 5.10;
\[ h \]  = full depth of beam or height of each zone in stress diagram;
\( E \) = tensile modulus;

\( E_c \) = compressive modulus;

\( E_{ci} \) = initial compressive modulus;

\( E_{cr} \) = postcrack tensile modulus;

\( F \) = force component in stress diagram;

\( f \) = stress at vertex in stress diagram;

\( f'c \) = uniaxial ultimate compressive strength;

\( K \) = flexural stiffness of uncracked beam;

\( k \) = neutral axis depth ratio;

\( L \) = clear span;

\( L_p \) = plastic length for crack localization in beams;

\( M \) = moment;

\( M_{bcr} \) = bilinear cracking moment;

\( M_{fail} \) = moment at failure;

\( M_{low} \) = moment at the lowest point before continue the 2\textsuperscript{nd} ascending curve;

\( M_{max} \) = maximum moment in moment curvature diagram;

\( M_{max1}, M_{max2} \) = local maximum moment 1 and 2 in moment curvature diagram;

\( M_{cr2} \) = reduced cracking moment;

\( M_e \) = moment in elastic range;

\( M_{pc} \) = moment in postcrack range;

\( M_{it} \) = moment at an intersection point;

\( P \) = total load applied to beam specimen;
\( V_f \) = fiber volume fraction;

\( X \) = distance from the support to the first point load in beam specimen;

\( y \) = moment arm from neutral axis to center of each force component;

\( \alpha \) = normalized transition strain \((\varepsilon_{trn}/\varepsilon_{cr})\);

\( \beta \) = normalized tensile strain at bottom fiber \((\varepsilon_{tbot}/\varepsilon_{cr})\);

\( \beta_{crit} \) = normalized critical tensile strain;

\( \beta_{tu} \) = normalized ultimate tensile strain \((\varepsilon_{tu}/\varepsilon_{cr})\);

\( \delta_{bcr} \) = deflection at bilinear cracking moment;

\( \delta_{u} \) = deflection at ultimate moment;

\( \gamma \) = normalized compressive modulus \((E_c/E)\);

\( \varepsilon \) = strain;

\( \varepsilon_c \) = compressive strain;

\( \varepsilon_{ctop} \) = compressive strain at top fiber;

\( \varepsilon_t \) = tensile strain;

\( \varepsilon_{tbot} \) = tensile strain at bottom fiber;

\( \varepsilon_{trn} \) = tensile strain at transition point;

\( \phi \) = curvature;

\( \phi_{bcr} \) = bilinear cracking curvature;

\( \phi_e \) = curvature in elastic range;

\( \phi_{pc} \) = curvature in postcrack range;

\( \phi_{it} \) = curvature at an intersection point of bilinear moment-curvature model;
\( \eta \) = normalized postcrack tensile modulus \( (E_{cr}/E) \);

\( \lambda \) = normalized compressive strain at top fiber \( (\varepsilon_{\text{top}}/\varepsilon_{cr}) \);

\( \lambda_{cu} \) = normalized ultimate compressive strain \( (\varepsilon_{cu}/\varepsilon_{cr}) \);

\( \lambda_{tu} \) = normalized compressive strain at top fiber when bottom fiber reaches ultimate tensile strain;

\( \mu \) = normalized constant postpeak tensile strength \( (\sigma_p/\sigma_{cr}) \) or \( (\sigma_{cst}/\sigma_{cr}) \);

\( \mu_{crit} \) = critical normalized postpeak tensile strength that defines deflection softening to hardening;

\( \sigma \) = stress;

\( \sigma_c \) = compressive stress;

\( \sigma_p \) = constant postpeak tensile strength;

\( \sigma_t \) = tensile stress;

\( \omega \) = normalized compressive yield strain \( (\sigma_{cy}/\varepsilon_{cr}E_c) \);

\( \xi \) = unloading factor in moment curvature diagram; and

\( \psi \) = interpolation parameter for normalized reduced cracking moment.

**Super Script**

\( (') \) = normalized with either moment or curvature at cracking;

**Sub Script**

\( 1, 2, 3 \) = stage 1, 2 and 3 according to the imposed normalized compressive strain at top fiber \( \lambda \);
1, 21, 22, 31, 32 = stage 1, 2.1, 2.2, 3.1, 3.2 according to the imposed normalized tensile strain at bottom fiber $\beta$;

t1, t2, t3 = tension zone 1, 2, 3;

c1, c2 = compression zone 1, 2;

cr = at cracking tensile strain;

cy = at compressive yield strain;

tu = at ultimate tensile strain;

cu = at ultimate compressive strain; and

$\infty$ = at infinite tensile or compressive strain.
Table 5.1. Force Component and Moment Arm to Neutral Axis at Each Stage of Normalized Top Compressive Strain ($\lambda$)

<table>
<thead>
<tr>
<th>Stage</th>
<th>$F'_{c2}$</th>
<th>$F'_{c1}$</th>
<th>$F'_{t1}$</th>
<th>$F'_{t2}$</th>
<th>$y'_{c2}$</th>
<th>$y'_{c1}$</th>
<th>$y'_{t1}$</th>
<th>$y'_{t2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \lambda \leq 1$</td>
<td>-</td>
<td>$\frac{\lambda}{4}$</td>
<td>$\frac{\lambda}{4}$</td>
<td>-</td>
<td>-</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>-</td>
</tr>
<tr>
<td>$1 &lt; \lambda \leq \omega$</td>
<td>-</td>
<td>$\frac{\lambda k}{2}$</td>
<td>$\frac{k}{2\lambda}$</td>
<td>$\mu - \frac{\mu (\lambda + 1)k}{\lambda}$</td>
<td>-</td>
<td>$\frac{2k}{3}$</td>
<td>$\frac{2k}{3\lambda}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\omega &lt; \lambda \leq \lambda_{cu}$</td>
<td>$\frac{\omega k(\lambda - \omega)}{\lambda}$</td>
<td>$\frac{\omega^2 k}{2\lambda}$</td>
<td>$\frac{k}{2\lambda}$</td>
<td>$\mu - \frac{\mu (\lambda + 1)k}{\lambda}$</td>
<td>$(\omega + \lambda)k$</td>
<td>$\frac{2\omega k}{3\lambda}$</td>
<td>$\frac{2k}{3\lambda}$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

Note that force components and their moment arms to neutral axis are written in normalized forms, $F_{xi}/(bhE_\varepsilon_c)$ and $y_{xi}/h$, respectively where $xi$ refers to $c1$, $c2$, $t1$ and $t2$. 
Table 5.2. Net force, Moment and Curvature for Each Stage of Normalized Top Compressive Strain ($\lambda$)

<table>
<thead>
<tr>
<th>Stage</th>
<th>Net Force</th>
<th>Moment</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \lambda \leq 1$</td>
<td>$-F_{c1} + F_{r1}$</td>
<td>$F_{c1}y_{c1} + F_{r1}y_{r1}$</td>
<td></td>
</tr>
<tr>
<td>$1 &lt; \lambda \leq \omega$</td>
<td>$-F_{c1} + F_{r1} + F_{r2}$</td>
<td>$F_{c1}y_{c1} + F_{r1}y_{r1} + F_{r2}y_{r2}$</td>
<td>$\frac{\lambda e_x}{kh}$</td>
</tr>
<tr>
<td>$\omega &lt; \lambda \leq \lambda_{cu}$</td>
<td>$-F_{c1} - F_{c2} + F_{r1} + F_{r2}$</td>
<td>$F_{c1}y_{c1} + F_{c2}y_{c2} + F_{r1}y_{r1} + F_{r2}y_{r2}$</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.3. Neutral Axis Depth Ratio, Normalized Moment and Curvature for Each Stage of Normalized Top Compressive Strain ($\lambda$)

<table>
<thead>
<tr>
<th>Stage</th>
<th>$k$</th>
<th>$M'$</th>
<th>$\phi'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \lambda \leq 1$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\lambda}{2k}$</td>
<td></td>
</tr>
<tr>
<td>$1 &lt; \lambda \leq \omega$</td>
<td>$\frac{2\mu \lambda}{\lambda^2 + 2\mu(\lambda + 1) - 1}$</td>
<td>$\frac{(2\lambda^3 + 3\mu\lambda^2 - 3\mu + 2)k^2}{\lambda^2} - 3\mu(2k - 1)$</td>
<td>$\frac{\lambda}{2k}$</td>
</tr>
<tr>
<td>$\omega &lt; \lambda \leq \lambda_{cu}$</td>
<td>$\frac{2\mu \lambda}{-\omega^2 + 2\omega(\omega + \mu) + 2\mu - 1}$</td>
<td>$\frac{(3\omega\lambda^2 - \omega^3 + 3\mu\lambda^2 - 3\mu + 2)k^2}{\lambda^2} - 3\mu(2k - 1)$</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.4. Details of the Mixes and Their Parameters Used in Simulation of Load-Deflection Response of Steel Fiber Reinforced Concrete

<table>
<thead>
<tr>
<th>Set</th>
<th>L</th>
<th>V_f</th>
<th>Test</th>
<th>L_p</th>
<th>E</th>
<th>ε_{cr}</th>
<th>ε_{cu}</th>
<th>μ</th>
<th>ω</th>
<th>ε_{cu}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm</td>
<td>%</td>
<td>mm</td>
<td>mm</td>
<td>MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H22</td>
<td>30</td>
<td>1.0</td>
<td>3P</td>
<td>300</td>
<td>N/A</td>
<td>25,400</td>
<td>116.2</td>
<td>0.015</td>
<td>0.44</td>
<td>9.8</td>
</tr>
<tr>
<td>H22*</td>
<td></td>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H3</td>
<td>30</td>
<td>1.5</td>
<td>3P</td>
<td>300</td>
<td>N/A</td>
<td>25,500</td>
<td>121.9</td>
<td>0.015</td>
<td>0.73</td>
<td>9.3</td>
</tr>
<tr>
<td>H3*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1</td>
<td>30</td>
<td>0.5</td>
<td>4P</td>
<td>750</td>
<td>N/A</td>
<td>25,400</td>
<td>110.6</td>
<td>0.015</td>
<td>0.24</td>
<td>10.3</td>
</tr>
<tr>
<td>H1*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H21</td>
<td>50</td>
<td>1.0</td>
<td>4P</td>
<td>750</td>
<td>N/A</td>
<td>25,400</td>
<td>116.2</td>
<td>0.015</td>
<td>0.83</td>
<td>9.8</td>
</tr>
<tr>
<td>H21*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.83</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that beam section size is 100x100 mm, compressive strength f_{c'}=34 MPa, assumed yield compressive stress σ_{cy} = 0.85*f_{c'}. Superscript (*) refers to the modified model to allow the prediction to match to the experimental result.
Table 5.5. Normalized Height of Compression and Tension Zones for Each Stage of
Normalized Tensile strain at Bottom Fiber ($\beta$)

<table>
<thead>
<tr>
<th>Normalized height</th>
<th>Stage 1</th>
<th>Stage 2.1</th>
<th>Stage 2.2</th>
<th>Stage 3.1</th>
<th>Stage 3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>and</td>
<td>$(0&lt;\beta \leq 1)$</td>
<td>$(1&lt;\beta \leq \alpha)$</td>
<td>$(1&lt;\beta \leq \alpha)$</td>
<td>$(\alpha&lt;\beta \leq \beta_t)$</td>
<td>$(\alpha&lt;\beta \leq \beta_{tu})$</td>
</tr>
<tr>
<td>and</td>
<td>$(0&lt;\lambda \leq \omega)$</td>
<td>$(0&lt;\lambda \leq \omega)$</td>
<td>$(\omega &lt; \lambda \leq \lambda_{cu})$</td>
<td>$(0&lt;\lambda \leq \omega)$</td>
<td>$(\omega &lt; \lambda \leq \lambda_{cu})$</td>
</tr>
</tbody>
</table>

\[
\frac{h_{c2}}{h} = \frac{k\beta - \omega(1-k)}{\beta}, \quad \frac{h_{c1}}{h} = \frac{\omega(1-k)}{\beta}, \quad \frac{h_{t1}}{h} = 1-k, \quad \frac{h_{t2}}{h} = \frac{(1-k)(\beta-1)}{\beta}, \quad \frac{h_{t3}}{h} = \frac{(1-k)(\beta-\alpha)}{\beta}
\]
Table 5.6. Normalized Stress at Vertices in the Stress Diagram for Each Stage of Normalized Tensile Strain at Bottom Fiber ($\beta$)

<table>
<thead>
<tr>
<th>Normalized stress</th>
<th>Stage 1</th>
<th>Stage 2.1</th>
<th>Stage 2.2</th>
<th>Stage 3.1</th>
<th>Stage 3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(0 &lt; \beta \leq 1)$</td>
<td>$(1 &lt; \beta \leq \alpha)$</td>
<td>$(1 &lt; \beta \leq \alpha)$</td>
<td>$(\alpha &lt; \beta \leq \beta_{tu})$</td>
<td>$(\alpha &lt; \beta \leq \beta_{cu})$</td>
</tr>
<tr>
<td>and</td>
<td>and</td>
<td>and</td>
<td>and</td>
<td>and</td>
<td>and</td>
</tr>
<tr>
<td>$(0 &lt; \lambda \leq \omega)$</td>
<td>$(0 &lt; \lambda \leq \omega)$</td>
<td>$(\omega &lt; \lambda \leq \lambda_{cu})$</td>
<td>$(0 &lt; \lambda \leq \omega)$</td>
<td>$(\omega &lt; \lambda \leq \lambda_{cu})$</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{f_{c2}}{E \varepsilon_{cr}} = \begin{cases} 
\omega \gamma & \text{Stage 1} \\
\omega \gamma & \text{Stage 2.1} \\
\omega \gamma & \text{Stage 2.2} \\
\omega \gamma & \text{Stage 3.1} \\
\omega \gamma & \text{Stage 3.2}
\end{cases}
\]

\[
\frac{f_{c1}}{E \varepsilon_{cr}} = \begin{cases} 
\frac{\gamma \beta k}{1-k} & \text{Stage 1} \\
\frac{\gamma \beta k}{1-k} & \text{Stage 2.1} \\
\omega \gamma & \text{Stage 2.2} \\
\frac{\gamma \beta k}{1-k} & \text{Stage 3.1} \\
\omega \gamma & \text{Stage 3.2}
\end{cases}
\]

\[
\frac{f_{t1}}{E \varepsilon_{cr}} = \begin{cases} 
\beta & \text{Stage 1} \\
1 & \text{Stage 2.1} \\
1 & \text{Stage 2.2} \\
1 & \text{Stage 3.1} \\
1 & \text{Stage 3.2}
\end{cases}
\]

\[
\frac{f_{t2}}{E \varepsilon_{cr}} = \begin{cases} 
- & \text{Stage 1} \\
1 + \eta(\beta - 1) & \text{Stage 2.1} \\
1 + \eta(\alpha - 1) & \text{Stage 2.2} \\
1 + \eta(\alpha - 1) & \text{Stage 3.1} \\
1 + \eta(\alpha - 1) & \text{Stage 3.2}
\end{cases}
\]

\[
\frac{f_{t3}}{E \varepsilon_{cr}} = \begin{cases} 
- & \text{Stage 1} \\
- & \text{Stage 2.1} \\
- & \text{Stage 2.2} \\
- & \text{Stage 3.1} \\
\mu & \text{Stage 3.2}
\end{cases}
\]
Table 5.7. Normalized Force Component for Each Stage of Normalized Tensile Strain at Bottom Fiber ($\beta$)

<table>
<thead>
<tr>
<th>Normalized Force Component</th>
<th>Stage 1</th>
<th>Stage 2.1</th>
<th>Stage 2.2</th>
<th>Stage 3.1</th>
<th>Stage 3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(0 &lt; \beta \leq 1)$</td>
<td>$(1 &lt; \beta \leq \alpha)$</td>
<td>$(1 &lt; \beta \leq \alpha)$</td>
<td>$(\alpha &lt; \beta \leq \beta_{cu})$</td>
<td>$(\alpha &lt; \beta \leq \beta_{cu})$</td>
</tr>
<tr>
<td>and and and and and and</td>
<td>$(0 &lt; \lambda \leq \omega)$</td>
<td>$(0 &lt; \lambda \leq \omega)$</td>
<td>$(\omega &lt; \lambda \leq \lambda_{cu})$</td>
<td>$(0 &lt; \lambda \leq \omega)$</td>
<td>$(\omega &lt; \lambda \leq \lambda_{cu})$</td>
</tr>
</tbody>
</table>

\[
\frac{F_{c2}}{bhE\varepsilon_{cr}} = \begin{align*}
- & - \quad \frac{\omega \gamma}{\beta} (\beta k + \omega k - \omega) \\
\end{align*}
\]

\[
\frac{F_{c1}}{bhE\varepsilon_{cr}} = \begin{align*}
\frac{\beta \gamma k^2}{2(1-k)} & \quad \frac{\beta \gamma k^2}{2(1-k)} & \quad \frac{\omega^2 \gamma}{2\beta} (1-k) & \quad \frac{\beta \gamma k^2}{2(1-k)} & \quad \frac{\omega^2 \gamma}{2\beta} (1-k) \\
\end{align*}
\]

\[
\frac{F_{c1}}{bhE\varepsilon_{cr}} = \begin{align*}
\frac{\beta}{2} (1-k) & \quad \frac{(1-k)}{2\beta} & \quad \frac{(1-k)}{2\beta} \\
\end{align*}
\]

\[
\frac{F_{t1}}{bhE\varepsilon_{cr}} = \begin{align*}
- & \quad \frac{(1-k)(\beta - 1)(\eta\beta - \eta + 2)}{2\beta} \\
\end{align*}
\]

\[
\frac{F_{t2}}{bhE\varepsilon_{cr}} = \begin{align*}
- & \quad \frac{(1-k)(\alpha - 1)(\eta\alpha - \eta + 2)}{2\beta} \\
\end{align*}
\]

\[
\frac{F_{t3}}{bhE\varepsilon_{cr}} = \begin{align*}
- & \quad \frac{(1-k)(\beta - \alpha) \mu}{\beta} \\
\end{align*}
\]
Table 5.8. Normalized Moment Arm of Force Component for Each Stage of Normalized Tensile Strain at Bottom Fiber (β)

<table>
<thead>
<tr>
<th>Normalized moment arm</th>
<th>Stage 1</th>
<th>Stage 2.1</th>
<th>Stage 2.2</th>
<th>Stage 3.1</th>
<th>Stage 3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0 &lt; β ≤ 1)</td>
<td>(1 &lt; β ≤ α)</td>
<td>(1 &lt; β ≤ α)</td>
<td>(α &lt; β ≤ β_{cu})</td>
<td>(α &lt; β ≤ β_{cu})</td>
</tr>
<tr>
<td>and and and and and</td>
<td>(0 &lt; λ ≤ ω)</td>
<td>(0 &lt; λ ≤ ω)</td>
<td>(ω &lt; λ ≤ λ_{cu})</td>
<td>(0 &lt; λ ≤ ω)</td>
<td>(ω &lt; λ ≤ λ_{cu})</td>
</tr>
</tbody>
</table>

| y_{c2} | - | - | \frac{βk + ω(1-k)}{2β} | - | \frac{βk + ω(1-k)}{2β} |
| \frac{y_{c1}}{h} | \frac{2}{3}k | \frac{2}{3}k | \frac{2}{3}ω(1-k) | \frac{2}{3}k | \frac{2}{3}ω(1-k) |
| \frac{y_{i1}}{h} | \frac{2}{3}(1-k) | \frac{2}{3}(1-k) | \frac{2}{3}(1-k) | \frac{2}{3}(1-k) |
| \frac{y_{i2}}{h} | - | \frac{2ηβ^2 - ηβ - η + 3β + 3}{3β(ηβ - η + 2)}(1-k) | \frac{2ηα^2 - ηα - η + 3α + 3}{3β(ηα - η + 2)}(1-k) |
| \frac{y_{i3}}{h} | - | - | - | \frac{(α + β)}{2β}(1-k) |
Table 5.9. Equilibrium of Force, Moment and Curvature for Each Stage of Normalized Tensile Strain at Bottom Fiber ($\beta$)

<table>
<thead>
<tr>
<th>Stage</th>
<th>Tension</th>
<th>Comp.</th>
<th>Force Equilibrium</th>
<th>Internal Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>($0 &lt; \beta \leq 1$)</td>
<td>($0 &lt; \lambda \leq \omega$)</td>
<td>$-F_{c1} + F_{t1}$</td>
<td>$F_{c1}y_{c1} + F_{t1}y_{t1}$</td>
</tr>
<tr>
<td>2.1</td>
<td>($1 &lt; \beta \leq \alpha$)</td>
<td>($0 &lt; \lambda \leq \omega$)</td>
<td>$-F_{c1} + F_{t1} + F_{t2}$</td>
<td>$F_{c1}y_{c1} + F_{t1}y_{t1} + F_{t2}y_{t2}$</td>
</tr>
<tr>
<td>2.2</td>
<td>($1 &lt; \beta \leq \alpha$)</td>
<td>($\omega &lt; \lambda \leq \lambda_{cu}$)</td>
<td>$-F_{c1} - F_{c2} + F_{t1} + F_{t2}$</td>
<td>$F_{c1}y_{c1} + F_{c2}y_{c2} + F_{t1}y_{t1} + F_{t2}y_{t2}$</td>
</tr>
<tr>
<td>3.1</td>
<td>($\beta &gt; \alpha$)</td>
<td>($0 &lt; \lambda \leq \omega$)</td>
<td>$-F_{c1} + F_{t1} + F_{t2} + F_{t3}$</td>
<td>$F_{c1}y_{c1} + F_{t1}y_{t1} + F_{t2}y_{t2} + F_{t3}y_{t3}$</td>
</tr>
<tr>
<td>3.2</td>
<td>($\beta &gt; \alpha$)</td>
<td>($\omega &lt; \lambda \leq \lambda_{cu}$)</td>
<td>$-F_{c1} - F_{c2} + F_{t1} + F_{t2} + F_{t3}$</td>
<td>$F_{c1}y_{c1} + F_{c2}y_{c2} + F_{t1}y_{t1} + F_{t2}y_{t2} + F_{t3}y_{t3}$</td>
</tr>
</tbody>
</table>

Note that Curvature = $\varepsilon_c/(kh)$
Table 5.10. Neutral Axis Depth Ratio, Normalized Moment and Curvature for Each Stage of Normalized Tensile Strain ($\beta$)

<table>
<thead>
<tr>
<th>Stage</th>
<th>$k$</th>
<th>$M'$</th>
<th>$\phi'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$k_1 = \begin{cases} \frac{1}{2} &amp; \text{for } \gamma = 1 \ -1 + \sqrt{\gamma} &amp; \text{for } \gamma &lt; 1 \text{ or } \gamma &gt; 1 \end{cases}$</td>
<td>$M'_1 = \frac{2\beta\left((\gamma - 1)k_1^3 + 3k_1^2 - 3k_1 + 1\right)}{1 - k_1}$</td>
<td>$\phi'_1 = \frac{\beta}{2(1 - k_1)}$</td>
</tr>
<tr>
<td>2.1</td>
<td>$k_{21} = \frac{\beta^2\gamma + D_{21} - \sqrt{\gamma^2\beta^4 + D_{21}\gamma^2}}{D_{21}}$</td>
<td>$M'<em>{21} = \frac{(2\beta\gamma + C</em>{21})k_{21}^3 - 3C_{21}k_{21}^2 + 3C_{21}k_{21} - C_{21}}{1 - k_{21}}$</td>
<td>$\phi'<em>{21} = \frac{\beta}{2(1 - k</em>{21})}$</td>
</tr>
<tr>
<td></td>
<td>$D_{21} = \eta\left(\beta^2 - 2\beta + 1\right) + 2\beta - \beta^2\gamma - 1$</td>
<td>$C_{21} = \frac{-2\eta\beta^3 + 3\eta\beta^2 - 3\beta^2 - \eta + 1}{\beta^2}$</td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>$k_{22} = \frac{D_{22}}{D_{22} + 2\omega\gamma\beta}$</td>
<td>$M'<em>{22} = \frac{(3\omega\gamma + C</em>{22})k_{22}^3 - 2C_{22}k_{22}^2 + C_{22}}{1 - k_{22}}$</td>
<td>$\phi'<em>{22} = \frac{\beta}{2(1 - k</em>{22})}$</td>
</tr>
<tr>
<td></td>
<td>$D_{22} = \eta\left(\beta^2 - 2\beta + 1\right) + 2\beta + \omega^2\gamma - 1$</td>
<td>$C_{22} = \frac{2\eta\beta^3 - 3\eta\beta^2 + 3\beta^2 - \omega^2\gamma + \eta - 1}{\beta^2}$</td>
<td></td>
</tr>
<tr>
<td>3.1</td>
<td>$k_{31} = \frac{D_{31} - \sqrt{\gamma^2\beta^4 + D_{31}\gamma^2}}{D_{31} - \beta^2\gamma}$</td>
<td>$M'<em>{31} = \frac{(C</em>{31} - 2\beta\gamma)k_{31}^3 - 3C_{31}k_{31}^2 + 3C_{31}k_{31} - C_{31}}{k_{31} - 1}$</td>
<td>$\phi'<em>{31} = \frac{\beta}{2(1 - k</em>{31})}$</td>
</tr>
<tr>
<td></td>
<td>$D_{31} = \eta\left(\alpha^2 - 2\alpha + 1\right) + 2\mu(\beta - \alpha) + 2\alpha - 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.2</td>
<td>$k_{32} = \frac{D_{32}}{D_{32} + 2\omega\gamma\beta}$</td>
<td>$M'<em>{32} = \frac{(C</em>{32} + 3\omega\gamma)k_{32}^3 - 2C_{32}k_{32}^2 + C_{32}}{1 - k_{32}}$</td>
<td>$\phi'<em>{32} = \frac{\beta}{2(1 - k</em>{32})}$</td>
</tr>
<tr>
<td></td>
<td>$D_{32} = \omega^2\gamma + \eta\alpha^2 + 2(\mu\beta - \eta\alpha - \mu\alpha + \alpha) + \eta - 1$</td>
<td>$C_{32} = \frac{3(\mu\beta^2 - \mu\alpha^2 - \eta\alpha^2 + \alpha^2) + 2\eta\alpha^3 + \eta - 1}{\beta^2}$</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 5.1. Relationship between uniaxial tension test and flexural test of fabric reinforced cement: (a) comparison of experimental responses; and (b) comparison of cumulative probability distributions for LOP, MOR, BOP and UTS.
Fig. 5.2. Strain-softening fiber reinforced concrete model: (a) tension model; and (b) compression model
Fig. 5.3. Stress strain diagram at 3 stages of normalized top compressive strain $\lambda$: (a) elastic for compression and tension ($0 < \lambda \leq 1$); (b) elastic for compression but nonlinear for tension ($1 < \lambda \leq \omega$); and (c) plastic for compression and nonlinear for tension ($\lambda > \omega$)
Fig. 5.4. Normalized moment curvature diagrams and their approximate bilinear models:

(a) deflection-hardening ($\mu > \mu_{\text{crit}}$); and (b) deflection-softening ($\mu < \mu_{\text{crit}}$)
Fig. 5.5. Comparison of normalized moment equations: (a) regression equation to determine the intersection point for normalized bilinear moment curvature diagram; (b) the normalized moment capacity between $M'_\infty$ and $M'_u$
Fig. 5.6. Three- and four-point bending test: (a) experimental setup; (b) moment distribution; (c) curvature distribution at bilinear cracking; (d) curvature distribution at ultimate moment for high normalized postpeak tensile strength ($\mu > \mu_{\text{crit}}$); (e) curvature distribution at ultimate moment for low normalized postpeak tensile strength ($\mu < \mu_{\text{crit}}$)
Fig. 5.7. Effect of normalized postpeak tensile strength $\mu$ to: (a) neutral axis depth ratio $k$; (b) normalized moment curvature diagram $M' - \phi'$
Fig. 5.8. Effect of compressive yield strength ratio $\omega$ to: (a) neutral axis depth ratio $k$; (b) normalized moment curvature diagram $M' - \phi'$.
Experiment (a) \( E = 25400 \text{ MPa}, \omega = 9.8 \)
\( \varepsilon_{tu} = 0.015, \quad \varepsilon_{cu} = 0.004 \)

\( \varepsilon_{cr} = 247 \mu \text{str}, \mu = 0.30 \)
\( \varepsilon_{cr} = 116 \mu \text{str}, \mu = 0.44 \)

Experiment (b)
Fig. 5.9. Tension models and the predicted load-deflection response of steel fiber reinforced concrete under three-point bending: (a) tension model for H22; (b) load-deflection response for H22; (c) tension model for H3; (d) load-deflection response for H3
(a) \[ E = 25400 \text{ MPa}, \quad \omega = 10.3 \]
\[ \varepsilon_{tu} = 0.015, \quad \varepsilon_{cu} = 0.004 \]

(b) \[ \varepsilon_{cr} = 178 \mu \text{str}, \quad \mu = 0.24 \]
\[ \varepsilon_{cr} = 111 \mu \text{str}, \quad \mu = 0.24 \]

Experiment

\[ E = 25400 \text{ MPa}, \quad \omega = 10.3 \]
\[ \varepsilon_{tu} = 0.015, \quad \varepsilon_{cu} = 0.004 \]
Fig. 5.10. Tension models and the predicted load-deflection responses of steel fiber reinforced concrete under four-point bending: (a) tension model for H1; (b) load-deflection response for H1; (c) tension model for H21; (d) load-deflection response for H21.
Fig. 5.11. Material models for homogenized fiber reinforced concrete: (a) compression model; and (b) tension model
Fig. 5.12. Stress strain diagram at different stages of normalized tensile strain at the bottom fiber ($\beta$):

(a) $0 < \beta < 1$ and $\lambda < \omega$;  
(b.1) $1 < \beta < \alpha$ and $\lambda < \omega$;  
(b.2) $1 < \beta < \alpha$ and $\omega < \lambda < \lambda_{cu}$; and  
(c.1) $\alpha < \beta < \beta_{tu}$ and $\lambda < \omega$;  
(c.2) $\alpha < \beta < \beta_{tu}$ and $\omega < \lambda < \lambda_{cu}$
Fig. 5.13. (a) Moment curvature diagram and crack localization rules; and (b) four point bending test
\[ \alpha = 15, \eta = -0.048 \]
\[ \alpha = 10, \eta = -0.074 \]
\[ \alpha = 5, \eta = -0.074 \]
\[ \alpha = 1.01, \eta = -67 \]

\[ E = 20 \text{ GPa} \]
\[ \varepsilon_{cr} = 0.0001 \]
\[ \mu = 0.33 \]
\[ \beta_{lu} = 150 \]
\[ \gamma = 1 \]
\[ \omega = 10 \]
\[ \lambda_{cu} = 30 \]
strain (mm/mm)

stress (MPa)

\( \mu = 0.99, \eta = -0.001 \)
\( \mu = 0.67, \eta = -0.037 \)
\( \mu = 0.33, \eta = -0.074 \)
\( \mu = 0.01, \eta = -0.11 \)

\( E = 20 \text{ GPa} \)
\( \varepsilon_{cr} = 0.0001 \)
\( \alpha = 10 \)
\( \beta_{tu} = 150 \)
\( \gamma = 1 \)
\( \omega = 10 \)
\( \lambda_{cu} = 30 \)
Fig. 5.14. Parametric study of a typical strain-softening material: the effect of parameters $\alpha$, $\mu$ and $\gamma$& $\omega$ to normalized moment curvature diagram.
\( \alpha = 15 \)
\( \alpha = 10 \)
\( \alpha = 5 \)
\( \alpha = 1.01 \)

\( E = 20 \) GPa
\( \varepsilon_{cr} = 0.0001 \)
\( \eta = 0.33 \)
\( \beta_{tu} = 150 \)
\( \mu = 0.01 \)
\( \gamma = 1 \)
\( \omega = 10 \)
\( \lambda_{cu} = 30 \)
Strain (mm/mm)

-20
-10
0
10
20

Stress (MPa)

η = 0.99
η = 0.67
η = 0.33
η = 0.01

E = 20 GPa
ε_{cr} = 0.0001
a = 10

β_{tu} = 150
μ = 0.01
γ = 1
ω = 10
λ_{cu} = 30

Normalized Moment, M'

Normalized Curvature, φ'

(d)
Fig. 5.15. Parametric study of a typical strain-hardening materials: the effect of parameters $\alpha$, $\eta$ and $\gamma$& $\omega$ to normalized moment curvature diagram.
Modified model (1.40\(\varepsilon_{cr}\))

- Fitted model
- Experiment

\[ \varepsilon_{cr} = 110.6 \mu \text{str}, \]
\[ b = 100 \text{ mm}, h = 100 \text{ mm}, \]
\[ L = 750 \text{ mm}, \]
\[ E = 25.4 \text{ GPa}, \]
\[ \eta = -0.760, \alpha = 2.0, \]
\[ \mu = 0.24, \beta_{tu} = 136, \]
\[ \gamma = 0.95, \omega = 10.8, \]
\[ \lambda_{cu} = 36.2 \]

---

Equivalent Flexural Stress (MPa)

- Modified model (1.40\(\varepsilon_{cr}\))
- Fitted model
- Experiment

\[ \lambda_{cu} = 36.2 \]
Fig. 5.16. Simulation of a steel fiber reinforced concrete (SFRC): (a) and (b) tension model and flexural response of mix H1 ($V_f = 0.5\%$); (c) and (d) tension model and flexural response of mix H21 ($V_f = 1.0\%$)

- $b = 100$ mm, $h = 100$ mm, $L = 750$ mm, $E = 25.4$ GPa, $\varepsilon_{cr} = 116.2$ $\mu$str, $\eta = -0.100$, $\alpha = 3.0$, $\mu = 0.80$, $\beta_{tu} = 129$, $\gamma = 0.95$, $\omega = 10.3$, $\lambda_{cu} = 34.4$
Fig. 5.17. Simulation of Engineered Cementitious Composite (ECC): (a) tension model; (b) flexural response
CHAPTER 6

FLEXURAL MODELING OF HYBRID REINFORCED CONCRETE

6.1. Introduction

Since introduction of fiber reinforced concrete (FRC) to the construction industry more than forty years ago, the demand for this material has steadily increased. The main areas of uses are primarily in slabs on grade and pavements, followed by tunnel linings and small percentage of precast and prestressed concrete products. Recently, based on the advancement of materials, free suspended elevated flat slabs made of steel fiber reinforced concrete were successfully implemented (Destrée 2006; Soranakom et al. 2007). In these cases, steel fibers replace conventional reinforcement in most parts to provide the flexural capacity; only small amount of reinforcement was needed in the column lines to prevent progressive failure.

Various aspects of fibers such as type, shape, aspect ratio and volume fraction have been used as design parameters to optimize tensile performance of FRC. As concrete technology continues to improve, it is expected to see more reinforced concrete members being replaced by FRC in order to save equipment, labor, and construction costs. In the situations that FRC alone cannot provide adequate strength, steel rebar or welded wire mesh may be added to increase the load carrying capacity. With this trend in concrete industries, several building codes provide guidelines on how to test and design with FRC materials (Fischer 2004; Teutsch 2004; Barr and Lee 2004; di Prisco et al. 2004; Massicotte 2004). Despite the research efforts have been heavily invested in the development of design guidelines, the inconsistencies among definition of material
properties, testing procedures, structural response, and design methodologies for flexural members are still controversial.

Due to its simplicity, the equivalent flexural strength at specific deflection 3 mm $f_{e,3}$ for a beam size 150 x 150 x 450 mm according to JCI-SF4 (1984) is currently being used in practices. The method assumes stress of concrete behavior elastically in pre and post crack stages, which greatly simplifies the complicate nature of stress distribution in concrete beam, at the same time; it can show the benefit of fibers in toughening the post crack response. One danger of this method is the equivalent strength can be misinterpreted as if it equals to the true tensile strength. In typical strain-softening materials such as concrete and rock, flexural strength overestimates uniaxial tensile strength (Wee et al. 2000; Chuang and Mai 1989). Thus, misinterpretation of strength used in the design can cause catastrophic failure.

Other more comprehensive models for FRC (Lim and Paramasivam 1987a,b; Lok and Pei 1998; Barros and Figueiras 1999; Vandewalle 2000 and 2002) are available in recent literatures. The most recognized one is the RILEM model proposed by Vandewalle (2003) that captures essential characteristics of FRC. Application of this method requires one to perform strain compatibility analysis of the layered beam section in order to obtain moment capacity, an approach that may not be routinely practical for general users. Alternatively, the simple strain-softening models for FRC presented in the previous chapter and their closed form solutions to predict flexural behavior is more attractive. The approach requires substitution of normalized material parameters in analytical equations to obtain moment-curvature response and flexural capacity and the material properties.
can be obtained by inverse analysis of load-deflection response of three- or four-point bending test.

In order to facilitate the use of discrete fibers, continuous reinforcement and a hybrid blend of these two systems, a unified flexural modeling of these three is developed in this chapter. The model is an extension of the strain-softening model for FRC, which incorporates of steel rebar model in formulation and allows unequal modulus in compression and tension. The derivation of complete moment curvature response is accomplished and the solutions are further simplified to obtain two analytical equations for predicting ultimate moment capacity and determining the minimum flexural reinforcement ratio.

6.2. Material Models

Fig. 6.1 presents the three distinct material models and the rectangular beam cross section used in derivation of moment-curvature diagram for modeling of flexural responses of FRC beams. Material parameters are described as a combination of normalized parameters and two intrinsic material parameters: tensile modulus $E$ and the cracking tensile strain $\varepsilon_{cr}$. An idealized tension model for FRC material is shown in Fig. 6.1(a). In the elastic range, the tensile stress increases linearly with a Young’s modulus $E$ up to the cracking tensile strength $\sigma_{cr}$ corresponding to the cracking tensile strain $\varepsilon_{cr}$. In the postpeak region, the strength drops to a postpeak tensile strength level $\sigma_p = \mu \varepsilon_{cr} E$ and remains constant before it is terminated at the ultimate tensile strain $\varepsilon_{tu} = \beta \varepsilon_{cr}$.

Fig. 6.1(b) shows the elastic-perfectly plastic model for compression response. The compressive modulus defined as $E_c = \gamma E$ can be different from the tensile modulus by a
factor of $\gamma$. The elastic-plastic model is defined by compressive yield strain $\varepsilon_{cy} = \omega \varepsilon_{cr}$, and hence compressive yield stress $\sigma_{cy} = \omega \gamma \varepsilon_{cr} E$, which is terminated at the ultimate compressive strain $\varepsilon_{cu}$. Fig. 6.1(c) presents the elastic-plastic steel model, which is similar to the compression model except that the yield strain $\varepsilon_{sy} = \kappa \varepsilon_{cr}$ and yield stress $f_{sy} = \kappa n \varepsilon_{cr} E$ are defined by different normalized parameters: $\kappa$ and $n$. There is no termination level specified for steel strain as it is assumed infinite plastic material. Likewise, geometrical parameters are defined as a combination of normalized parameters and beam dimensions: width $b$ and full depth $h$. Fig. 6.1(d) shows a beam cross section that contains an area of steel $A_s = \rho g bh$ at the reinforced depth $d = \alpha h$. It should be noted that the reinforcement ratio $\rho_g$ here is defined per gross sectional area $bh$, not an effective area $bd$ as normally used in reinforced concrete nomenclature. The material models for tension and compression of FRC and the model for steel rebar can be presented in mathematical forms as follows:

$$\sigma_i (\varepsilon_i) = \begin{cases} E \varepsilon_t & 0 \leq \varepsilon_i \leq \varepsilon_{cr} \\ \mu E \varepsilon_{cr} & \varepsilon_{cr} < \varepsilon_i \leq \varepsilon_{tu} \\ 0 & \varepsilon_i > \varepsilon_{tu} \end{cases} \quad (6.1)$$

$$\sigma_c (\varepsilon_c) = \begin{cases} E_c \varepsilon_c & 0 \leq \varepsilon_c \leq \varepsilon_{cy} \\ E_c \varepsilon_{cy} & \varepsilon_{cy} < \varepsilon_c \leq \varepsilon_{cu} \\ 0 & \varepsilon_c > \varepsilon_{cu} \end{cases} \quad (6.2)$$

$$f_s (\varepsilon_s) = \begin{cases} E_s \varepsilon_s & 0 \leq \varepsilon_s \leq \varepsilon_{sy} \\ E_s \varepsilon_{sy} & \varepsilon_s > \varepsilon_{sy} \end{cases} \quad (6.3)$$
In order to derive the closed form solutions for moment–curvature response in non-dimensional forms, the strains, modulus, strengths and reinforcing depth defined in Figs. 6.1(a-d) are normalized with respect to cracking tensile strain $\varepsilon_{cr}$, tensile modulus $E$, cracking tensile strength $\sigma_{cr} = \varepsilon_{cr} E$ and full depth of beam, respectively.

$$\beta_{tu} = \frac{\varepsilon_{tu}}{\varepsilon_{cr}}; \quad \omega = \frac{\varepsilon_{cy}}{\varepsilon_{cr}}; \quad \lambda_{cu} = \frac{\varepsilon_{cu}}{\varepsilon_{cr}}; \quad \kappa = \frac{\varepsilon_{sy}}{\varepsilon_{cr}}$$

(6.4)

$$\gamma = \frac{E_c}{E}; \quad n = \frac{E_s}{E}$$

(6.5)

$$\mu = \frac{\sigma_p}{E \varepsilon_{cr}}$$

(6.6)

$$\alpha = \frac{d}{h}$$

(6.7)

In addition, the normalized tensile strain at the bottom fiber $\beta$ and compressive strain at the top fiber $\lambda$ are defined as

$$\beta = \frac{\varepsilon_{bot}}{\varepsilon_{cr}}; \quad \lambda = \frac{\varepsilon_{top}}{\varepsilon_{cr}}$$

(6.8)

6.3. Derivation of Moment-Curvature Response

In derivation of moment curvature diagram for a rectangular beam cross section with a width of $b$ and depth of $h$, containing an area of steel at depth $d$, the Kirchhoff hypothesis of plane section remaining plane is assumed. By applying linear strain distribution across the depth, ignoring shear deformation, and using material models described in Figs. 6.1(a - c), the stress diagrams across the depth can be drawn as shown on the right of Fig. 6.2. A normalized compressive strain at the top concrete fiber $\lambda$ is used as an
independent variable to incrementally impose flexural deformation, which is defined in three stages. The first stage \(0 < \lambda \leq \lambda_{R1}\) corresponds to the compressive strain ranging from zero to the point where the tensile strain at the bottom fiber reaches the cracking tensile strain, which is the end of elastic response in region 1. Stage 2 \(\lambda_{R1} < \lambda \leq \omega\) corresponds to the compressive strain continuing in the elastic range but the tensile strain enters the postpeak region. Finally, stage 3 \(\omega < \lambda \leq \lambda_{cu}\) corresponds to the compressive strain is in plastic range and the tensile strain is in postpeak range. For stage 2 and 3 there are two possible scenarios: the tensile steel is either elastic or yielding. These cases depend on the material properties used in cross section and they will be treated in subsequent sections.

According to three stages of stress diagrams in Fig. 6.2, the heights of compression and tension zones are normalized with respect to the beam depth \(h\), magnitudes of stresses at the vertices are normalized with respect to the cracking tensile strength \(E\varepsilon_{cr}\) and their normalized forms are presented in Tables 6.1 and 6.2, respectively. The area and centroid of stress blocks in each zone represent the force components and lines of action. Their normalized values with respect to cracking tensile force \(bhE\varepsilon_{cr}\) and beam depth \(h\) are presented in Tables 6.3 and 6.4 respectively.

Table 6.5 shows the steps in determination of net section force, moment, and curvature at each stage of normalized compressive strain, \(\lambda\). The net force is obtained as the difference between the tension and compression forces, equated to zero for internal equilibrium, and solved for the neutral axis parameter \(k\). When steel is elastic in stages 1, 2.1 and 3.1, the expressions for net force are in the quadratic forms and result in two
possible solutions for $k$. With a large scale of numerical tests covering a practical range of material parameters, only one solution for $k$ yields the valid value in the range $0 < k < 1$.

Only stage 1, the singularity of $k_1$ is found when $\gamma = 1$, thus additional expression for $k_1$ is derived by taking the limit as $\gamma \rightarrow 1$. On the other hand, when steel is yielding in stage 2.2 or 2.3, it results in only one solution for $k$. The valid solutions of $k$ for all stages (1, 2.1, 2.2, 3.1 and 3.2) are presented together in Table 6.6. The internal moment is then obtained using the force components and their location with respect to the neutral axis. The curvature is determined as the ratio of the compressive strain at top fiber ($\varepsilon_{\text{top}} = \lambda \varepsilon_{cr}$) to the depth of neutral axis $kh$. The effective flexural stiffness is defined as the ratio of the moment to the curvature at any given imposed compressive strain level. Finally, the moment $M$, curvature $\phi$ and stiffness $K$ for each stage are normalized with respect to their values at cracking of plain FRC $M_{cr}$, $\phi_{cr}$ and $K_{cr}$ respectively and their closed form solutions $M'$, $\phi'$ and $K'$ are presented in Table 6.6.

$$M = M' M_{cr}; \quad M_{cr} = \frac{1}{6} bh^2 E \varepsilon_{cr}$$  \hspace{1cm} (6.9)

$$\phi = \phi' \phi_{cr}; \quad \phi_{cr} = \frac{2 \varepsilon_{cr}}{h}$$  \hspace{1cm} (6.10)

$$K = K' K_{cr}; \quad K_{cr} = \frac{1}{12} bh^3$$  \hspace{1cm} (6.11)

Since the compressive modulus $E_c$ may not be equal to the tensile modulus $E$, the normalized compressive strain corresponding to end of elastic region 1 ($\lambda_{R1}$) must be determined from the strain gradient diagram shown in Fig. 6.2(a).
By substituting $k_1$ from Table 6.6 for $k$ in Eq. (6.12) and solving for $\lambda_{R1}$, one obtains

$$\lambda_{R1} = \frac{\epsilon_{cr}}{kh} = \frac{\epsilon_{cr}}{(1-k)h}$$

(6.12)

The yield condition for tensile steel can be checked by first assuming that it yields and then using $k_{22}$ or $k_{32}$ in Table 6.6 for $k$ in Eq. (6.14) to calculate the steel strain $\epsilon_s$

$$\epsilon_s = \frac{\alpha-k}{k} \lambda \epsilon_{cr}$$

(6.14)

If $\epsilon_s$ is greater than $\epsilon_{sy}$, the assumption is correct, otherwise steel has not yielded and one has to use $k_{21}$ or $k_{31}$. Once, the neutral axis parameter $k$ and the applicable case are determined, the appropriate expressions for moment, curvature and stiffness in Table 6.6 and Eqs. (6.9) - (6.11) are then used to generate moment curvature response and its flexural stiffness.

To avoid compression failure occurring in ultimate stage, the steel used in flexural members must be less than the balanced reinforcement ratio $\rho_{g, bal}$, representing the simultaneous achievement of concrete compressive strain at failure ($\epsilon_c = \epsilon_{cu}$) with the steel reaching its yield limit ($\epsilon_s = \epsilon_{sy}$). The strain gradient in stage 3.2 of Fig. 6.2(c) representing a compressive strain in plastic range and tensile strain is in the postpeak region is used to derive the balance reinforcement ratio.
\[
\frac{\lambda_{cu} \epsilon_{cr}}{kh} = \frac{\kappa \epsilon_{cr}}{(\alpha - k)h}
\] (6.15)

By substituting \( \lambda_{cu} \) in the expression for \( k_{32} \) in Table 6.6 and then use it for \( k \) in Eq. (6.15), one can solve for the balance reinforcement ratio as

\[
\rho_{g, bal} = \frac{2 \mu (\lambda_{cu} (\alpha - 1) + \alpha - \kappa) + \alpha \gamma \omega (2\lambda_{cu} - \omega) - \alpha}{2n\kappa (\lambda_{cu} + \kappa)}
\] (6.16)

### 6.4. Parametric Studies of Material Parameters

Parametric studies of two main reinforcing parameters: postpeak tensile strength and reinforcement ratio were conducted to demonstrate the variations in the location of neutral axis, moment curvature response, and stiffness degradation of a beam section as flexural deformation increases. These three material responses are presented as normalized quantities with respect to the values at cracking of plain FRC. Fig. 6.3 shows typical material models for steel fiber reinforced concrete (SFRC) and steel rebar used in the parametric studies. Two intrinsic material parameters: tensile modulus \( E \) of 24 GPa and cracking tensile strain \( \epsilon_{cr} \) of 125 \( \mu \)str were used. Other normalized parameters for tension and compression models of SFRC were: ultimate tensile strain \( \beta_{tu} = 160 \), compressive modulus \( \gamma = 1 \), compressive yield strain \( \omega = 8.5 \) and ultimate compressive strain \( \lambda_{cu} = 28 \). The normalized parameters for steel rebar were: modulus ratio \( n = 8.33 \), steel yield strain \( \kappa = 16 \) and reinforcing depth \( \alpha = 0.8 \). In the study, all parameters were kept constant and the postpeak tensile strength parameter \( \mu \) was varied from 0.00 to 1.00; the reinforcement ratio \( \rho_g \) was varied from 0.0 to 0.03.
Fig. 6.4 shows the change of neutral axis depth ratio \( k \) as the increase of normalized compressive strain at top fiber \( \lambda \). For plain FRC system \((\mu \geq 0, \rho_g = 0)\), Fig. 4(a) shows the neutral axis starts at 0.5 as one would expect for the material that has equal compressive and tensile modulus \((\gamma = 1)\), and then drops, at different rates, depending on the level of postpeak tensile strength parameter \( \mu \). For brittle material represented by \( \mu = 0.00 \), \( k \) instantaneously drops to zero after initiation of cracking. As \( \mu \) increases from 0.00 to 1.00, the rate of decrease in neutral axis \( k \) becomes slower. A very ductile FRC with elastic–plastic tensile behavior defined as \( \mu = 1.00 \) yields the maximum value of \( k = 0.12 \) at the normalized ultimate compressive strain \( \lambda_{cu} \) of 28. For conventional reinforced concrete system \((\mu = 0, \rho_g > 0)\), Fig. 6.4(b) shows the effect of reinforcement ratio to the change of neutral axis. It can be seen that with the presence of reinforcement in plain concrete \((\mu = 0)\), the initial value of \( k \) is higher than 0.5 because the compressive zone in concrete section must be increased to balance with the tension force contributed from both concrete and steel. For plain concrete without any reinforcement \((\mu = \rho_g = 0)\), \( k \) drops sharply to zero after cracking. When 1% of reinforcement is added \((\rho_g = 0.01)\), \( k \) drops at a much slower rate and equal to 0.19 at \( \lambda_{cu} = 28 \), which is higher than \( k = 0.12 \) of the ductile FRC \((\mu = 1.00)\). At higher levels of \( \rho_g \) between 0.02 and 0.03, the \( k \) initially drops, then increases and the final value is relatively high compared to the starting value. Fig. 6.4(c) shows the effect of increasing postpeak tensile strength for a fixed reinforcement ratio \( \rho_g = 0.01 \). In this case, the responses are in between the responses of the two systems, but closer to those of plain FRC. The last Fig. 6.4(d) shows the effect of increasing reinforcement ratio for a fixed level of postpeak tensile strength \( \mu = 0.33 \). The
responses are slightly higher than those using reinforcement without postpeak strength in Fig. 6.4(b).

Fig. 6.5 shows the effect of parameters $\mu$ and $\rho_g$ to the flexural capacity represented by normalized moment curvature diagram. Fig. 6.5(a) shows that the moment curvature response of plain FRC system when postpeak tensile strength increases from brittle ($\mu = 0$) to ductile ($\mu = 1$). It should be noted that at $\mu = 0.33$ (close to $\mu_{\text{crit}} = 0.345$ in the conclusion of the previous chapter), the flexural response is almost elastic-perfectly plastic, in which the deflection-softening starts to shift to deflection-hardening. Using elastic-plastic tensile response ($\mu = 1$) of FRC yields an upper bound normalized moment capacity of 2.7. When compare FRC to the more efficient reinforced concrete system that utilizes steel rebar as a main reinforcement [Fig. 6.5(b)], the maximum normalized moment capacity of 5.8 can be achieved by using $\rho_g$ only 0.01. It can be noticed that as $\rho_g$ increases, the response changes from ductile under-reinforced to brittle over-reinforced section. Fig. 6.5(c) reveals the response when varying postpeak tensile strength for a fixed amount of reinforcement ratio ($\rho_g = 0.01$). The responses are similar to the curve using $\rho_g = 0.01$ in Fig. 6.5(b) plus additional strength from postpeak tensile strength ($\mu = 0.00 – 1.00$) as previously shown in Fig. 6.5(a). This hybrid system provides intermediate strength between the weaker plain FRC and the stronger conventional reinforced concrete. Fig. 6.5(d) reveals that there is little benefit to use postpeak tensile strength $\mu$ of 0.33 or less to the reinforced concrete system as the moment capacity slightly increases from the reinforced concrete without any fibers [Fig. 6.5(b)]. It should be pointed out that ductility of each curve shown in Fig. 6.5(d) is less than that in Fig.
6.5(b). This is due to the fact that the tensile capacity is increased while the ultimate compressive strain was fixed to the same level \((\varepsilon_{cu} = 0.0035)\). In actual concrete mixtures, using discrete fibers to increase postpeak tensile strength also enhances its ultimate compressive strain and may yield more ductile responses.

Fig. 6.6 shows the effect of \(\mu\) and \(\rho_g\) to the normalized secant stiffness \(K'\). As one would expect the stiffness degradation of plain FRC is quite severe. Fig. 6.6(a) shows that the initial normalized stiffness is 1.0 when the normalized curvature is in elastic range \(\phi' < 1.0\) and soon rapidly degrades after the section cracks \((\phi' > 1.0)\); the lower \(\mu\), the faster the stiffness decay rate. For reinforced concrete systems, Fig. 6.6(b) shows that the initial elastic stiffness is higher than 1.0, which is due to the presence of steel in homogenized material. After cracking, all postcrack stiffnesses decrease relatively at the same rate; but, more reinforcement ratio results in stiffer postcrack responses. Fig. 6.6(c) shows that the effect of using postpeak tensile strength at constant \(\rho_g\) of 0.01 results in slower stiffness degradation than those of plain FRC system [Fig. 6.6(a)] but faster than those of conventional reinforced concrete system [Fig. 6.6(b)]. Again using reinforcement with a fixed postpeak tensile strength \(\mu\) of 0.33 in Fig. 6.6(d) shows the responses just slightly stiffer than those of conventional reinforced concrete [Fig. 6.6(b)].

6.5. Design Charts

The ultimate moment capacity as a function of postpeak tensile strength and reinforcement ratio can be presented as a convenient design chart for any combination of concrete and steel properties used in a beam section. It is assumed that, in the ultimate stage, the concrete compressive strain at top fiber reaches its limiting value
(λ = λ_{cu}). The yielding of steel can be identified by comparing the reinforcement used to
the reinforcement ratio at balance failure as defined by Eq. (6.16). Once steel condition is
determined, appropriate expressions for neutral axis \( k_{31} \) or \( k_{32} \) are then used to calculate
the ultimate moment capacity \( M'_{31} \) or \( M'_{32} \).

Fig. 6.7 presents a design chart for a beam cross section using concrete and steel
models described in the parametric studies. The material parameters are also provided in
the chart. It can be seen that the normalized moment capacity is strongly dependent on
the amount of reinforcement ratio whereas the postpeak tensile strength provides extra
capacity. For under-reinforced sections, which are shown by the curves below the
balance failure points (\( \rho_g \leq \rho_{g,bal} \)), the moment capacity increases almost proportionally to
the reinforcement ratio. When \( \rho_g \) exceeds \( \rho_{g,bal} \), the strength of all curves slowly increase
as the added steel does not reach yield strength. To design flexural members with this
design chart, the ultimate moment \( M_u \) due to factored load must be determined and then
normalized with the cracking moment of the plain FRC \( M_{cr} \) to obtain normalized required
ultimate moment capacity \( M'_u \). The chart is then used to select any combination of
normalized postpeak tensile strength and reinforcement ratio that provides sufficient
strength for \( M'_u \), with reasonable safety factor.

6.5.1. Coefficient of Resistance

It is customary for structural engineers to use coefficient of resistance \( R \), presented as
design charts in reinforced concrete textbooks such as MacGregor (1997), to determine a
beam size for a given moment.

\[
M_n = Rbd^2
\]  
(6.17)
It can be shown that the proposed normalized moment design chart in Fig. 6.7 is equivalent to the well established $R$-chart. For single under-reinforced concrete design, the nominal moment capacity $M_n$ according to ACI method is

$$M_n = f'_c \Omega (1 - 0.59 \Omega) bd^2, \quad \text{and}$$

$$\Omega = \rho \frac{f_{sy}}{f'_c}$$

where $d$ is the effective depth, $\Omega$ is the mechanical reinforcement ratio, $\rho = A_s / bd$ is the reinforcement ratio per effect area $bd$. By comparing Eq. (6.17) to (6.18), the nominal coefficient of resistance $R$ based on ACI stress block can be presented as

$$R = f'_c \Omega (1 - 0.59 \Omega)$$

(6.20)

This Eq. (6.20) has an applicable range up to the balance failure, given by

$$\rho_{bal} = \frac{0.85 \beta_1 f'_c}{f_{sy} \left( \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{sy}} \right)}, \quad \text{and}$$

$$\beta_1 = 1.09 - 0.008 f'_c; \quad 0.65 \leq \beta_1 \leq 0.85$$

(6.21)

(6.22)

The proposed normalized moment design chart in Fig. 6.7 can be converted to the R-chart by using the relation of normalized moment to cracking moment [Eq. (6.9)] and reinforcing depth to full depth $\alpha = d/h$ [Eq. (6.7)]

$$R = \frac{M' \left( \lambda_{cu} \right)}{bd^2} = \frac{M' \left( \lambda_{cu} \right) M_{cr}}{bd^2} = \frac{M' \left( \lambda_{cu} \right)}{bd^2} \frac{1}{6} bh^2 \varepsilon_{cr} E = \frac{\varepsilon_{cr} E}{6 \alpha^2} M' \left( \lambda_{cu} \right)$$

(6.23)

Thus, $R$ is related to normalized moment $M' \left( \lambda_{cu} \right)$ by a factor of $\varepsilon_{cr}E/(6\alpha^2)$. 
To obtain compression parameters, the parabolic compressive stress-strain curve of Hognestad (1951) up to the ultimate strain $\varepsilon_{cu} = 0.003$ is set equivalent to the elastic perfectly plastic compression model, and used to obtain the relationship of the equivalent areas under curves:

$$
\varepsilon_{cu} \int_0^2 \left( \frac{\varepsilon_c}{\varepsilon_{c0}} \right)^2 d\varepsilon_c = \frac{1}{2} \varepsilon_{cy} \sigma_{cy} + (\varepsilon_{cu} - \varepsilon_{cy}) \sigma_{cy}
$$

By substituting $\varepsilon_{cu} = 1.5 \varepsilon_{c0}$ and $\sigma_{cy} = 0.85 f'_{c}$ in Eq.(6.24), the compressive yield strain $\varepsilon_{cy}$ and compressive modulus $E_c$ can be estimated as:

$$
\varepsilon_{cy} = \frac{6}{17} \varepsilon_{c0}; \quad E_c = \frac{\sigma_{cy}}{\varepsilon_{cy}} = \frac{289 f'_{c}}{120 \varepsilon_{c0}}
$$

The coefficient of resistance defined by the ACI approach [Eq.(6.20)] and the proposed method [Eq.(6.23)] were compared for three concrete strengths $f'_{c} = 30, 43$ and 55 MPa and two grades of steel 275 and 415 MPa with Young modulus $E_s$ of 200 GPa. The tensile modulus for concrete in MPa was estimated by the relationship $E = 4,735(f'_{c})^{0.5}$. Other parameters were assumed in typical range: cracking tensile strain $\varepsilon_{cr} = 0.0001$, the compressive strain at peak stress $\varepsilon_{c0} = 0.002$, the postpeak tensile strength for plain concrete $\mu = 0$ and the normalized depth of steel reinforcement $\alpha = 0.8$.

Figs. 6.8(a and b) compare the $R$ values for two grade of steel: 275 MPa and 415 MPa. The $R$ computed by ACI approach is shown by symbol plot while the $R$ computed by the proposed method is shown in continuous curves. Both curves show excellent agreement for the reinforcement ratio up to the ACI balance failure [Eq. (6.21)]. These point are generally lower than the balance failure points of the proposed method as marked by the
circle symbol defined by Eq. (6.16). The discrepancy between these two balance failures is due to the fact that ACI approach uses conservative empirical parameter $\beta_1$ [Eq.(6.22)] in the calculation of the reinforcement ratio at balance failure while the proposed method uses the analytical expression defined in Eq. (6.16). Note that $\rho = \rho_g/\alpha$ according to the relationship $\alpha = d/h$. It can be seen that the applicable range of the $R$ by ACI approach is terminated at the balance failure whereas the current method can predict a wider range in both under-reinforced and over reinforced beam sections.

6.6. Simplified Design Equation for Moment Capacity

In order to develop design guidelines, simple equations for determination of the moment capacity are desirable. It is noticed that the moment curvature responses of under-reinforced section in parametric studies [Fig. 6.5] show ductile behavior, in which the moment will approach a limiting value at infinity $(\lambda \to \infty)$ and the magnitude is relatively close to the moment at ultimate compressive strain failure $(\lambda = \lambda_{cu})$. Thus, the ultimate moment $M_u$ can be reasonably approximated by the moment at infinite compressive strain $M_{\infty}$ for under-reinforced section, identified by using $\rho_{g, bal}$ presented in Eq. (6.16). Normalized moment at infinite $M'_{\infty}$ can be found by substituting the expression for $k_{32}$ into the $M'_{32}$ in the same Table 6.6, followed by taking the limit of $\lambda$ to $\infty$, which results in

$$M'_{\infty} = \frac{6\rho_g n\kappa (\mu\alpha - \mu + \alpha\gamma) + 3(\gamma\omega\mu - (\rho_g n\kappa)^2)}{\gamma\omega + \mu}$$

(6.26)
For verification purpose, a set of parameters covering practical range of materials and locations of steel as shown in Table 6.7 were used in the expression for $k_{32}$ and $M'_{32}$ in Table 6.6 and the simplified Eq. (6.26). Table 6.7 presents 9 independent parameters ($f'_c, \sigma_{cr}, \mu, \sigma_{cy}, \gamma, e_{cu}, E_s, f_{sy}, \alpha$), each varying in 1 to 4 levels, which will generate $3 \times 3 \times 4 \times 3 \times 3 \times 1 \times 2 \times 3 = 5832$ combinations, respectively. The remaining 6 dependent parameters ($E, \varepsilon_{cr}, \omega, \lambda_{cu}, n, \kappa$) were calculated from the first 9 independent parameters by the relationship given in the Table 6.7.

Fig. 6.9 shows the comparison of predicted moment capacity between the original and simplified equations. As can be seen in Fig. 6.9(a) the ultimate moment $M'_{u}$ can be very well approximated by $M'_{\infty}$ as their solutions for 5,832 cases are in the equality line. Fig. 6.9(b) shows the reliability of the simplified equation which $M'_{\infty}/M'_{u}$ is plotted with respect to $\rho_{g}/\rho_{g,ba}$. The predicted ratio $M'_{\infty}/M'_{u}$ is varied in a narrow range between 1.00 and 1.054, with the mean and standard deviation of 1.005 and 0.0062, respectively. This implies that the simplified equation yields slightly unconservative ultimate moment capacity. More scatter in the predictability of the model is observed as reinforcement ratio increases from zero to balance failure. This is however expected since the simplification is based on the ideal ductile failure of under reinforced section.

6.7. Minimum Flexural Reinforcement Ratio

In serviceability stage, a reinforced concrete beam can fail abruptly if its post crack strength is less than the cracking moment of unreinforced concrete section computed from modulus of rupture. In order to prevent such failures, a minimum level of reinforcement must be provided to strengthen the post crack capacity of the designed
members at least equal to the cracking moment. This leads to the definition of minimum reinforcement specified by ACI Section 10.5.

Using the same criteria as ACI, the analytical expression for minimum reinforcement can be derived explicitly by setting the normalized moment at infinity $M'_{\infty}$ defined in Eq. (6.26) to the normalized cracking moment $M_{cr}' = 1$, resulting in a quadratic equation that has two solutions for the minimum reinforcement. With rigorous numerical testing of the solutions, the first one yields $\rho_{g, min} \leq \rho_{g, bal}$ while the other yields $\rho_{g, min} > \rho_{g, bal}$. The first inequality is the valid solution for the minimum reinforcement, which can be presented as

$$\rho_{g, min} = \frac{3G_1 - \sqrt{G_2 + 9G_1^2}}{3n\kappa}, \quad (6.27)$$

where $G_1 = \mu(\alpha - 1) + \alpha \gamma \omega$ and $G_2 = 3(\gamma \omega (3 \mu - 1) - \mu)$

In verification of the simplified Eq. (6.27), the same material parameters and steel locations given previously in Table 6.7 were used again, except that $\mu$ was varied in lower levels of 0.0, 0.1, 0.2, 0.3 and 0.4 since the values greater than $\mu_{crit} = 0.345$ will cause postcrack deflection-hardening; thus, negating minimum reinforcement. The same 5,832 combinations of parameters were generated and used as the input to calculate the minimum reinforcement. Fig. 6.10(a) shows that using the minimum reinforcement as calculated by Eq. (6.27) results in the normalized ultimate moment $M'_u$ and $M'_{\infty}$ predicted by the original and simplified equation in between 0.993 and 1.000, which is close to the theoretical value of 1.0; in other words, moment capacity does not drop after cracking and it remains constant equal to the ultimate moment. Fig. 6.10(b) plots the
required minimum reinforcement ratio vs. the postpeak tensile strength parameter $\mu$. The trend shows that as the postpeak tensile strength $\mu$ increases, the required minimum reinforcement $\rho_{g,\text{min}}$ decreases, and when $\mu$ approaches $\mu_{\text{crit}}$ of 0.345, no reinforcement is required as expected.

The proposed minimum reinforcement ratio defined in Eq. (6.27) was then compared to the equations used by ACI (2005), Vandewalle (2003), Shehata et al. (2003) and experimental results of Bosco et al (1989,1990). For uniformity of presentation, the minimum reinforcement ratio per effective area ($\rho_{\text{min}} = A_{s,\text{min}}/bd$) was used in the comparisons. According to ACI (2005), the minimum reinforcement ratio is empirically stipulated as a function of concrete compressive strength and steel-yield strength

$$\rho_{\text{min}1} = 0.25 \frac{f_c}{f_{sy}} \text{ but not less than 1.4} \quad (6.28)$$

Shehata et al. (2003) incorporated the flexural tensile strength suggested by CEB-FIP MC90 (1990) that captures the beam size effect in their derivation for the minimum flexural reinforcement ratio

$$\rho_{\text{min}2} = 0.005 \frac{f_{ck}^{0.67}}{f_{sy}} \left(1 + 1.5 \frac{h}{100}\right)^{0.7} \left(h/100\right)^{0.7} \quad (6.29)$$

where $f_{ck}$ is the cube concrete compressive strength. According to Eq. (6.27), the minimum reinforcement ratio $\rho_{g,\text{min}}$ is considered per gross section. Thus, it must be divided by the normalized depth of reinforcement $\alpha$ to yield the ratio per effective area.

$$\rho_{\text{min}3} = \frac{\rho_{g,\text{min}}}{\alpha} \quad (6.30)$$
Vandewalle (2003) proposed the minimum reinforcement to be used in the RILEM method as shown in Fig. 6.11. It can be considered as an extension of the Eurocode 2 (1992) by adding the average residual flexural tensile strength \( f_{Rm,1} \) term, which results in the following form.

\[
A_{s,\text{min}} = \left( k_c k_e k_p f_{fct,ef} - 0.45 f_{Rm,1} \right) \frac{A_{ct}}{f_s}
\]  

(6.31)

where \( k_c = 0.4 \) for pure bending; \( k_e \) is a coefficient accounting for the non uniform self equilibrating stress and defined as 1.0 for a depth less than 300 mm and 0.65 for a depth greater than 800 mm, and linearly varying between 300 mm and 800 mm. Coefficient \( k_p \) is for the prestressing effect and is ignored here \((k_p = 1)\). Parameter \( f_{fct,ef} \) is the tensile strength of FRC and it is assumed to be equal to the cracking tensile strength \( \sigma_{cr} \) for a fair comparison to the proposed model. The term \( 0.45 f_{Rm,1} \) is equal to second tensile stress point \( \sigma_2 \) defined in Fig. 6.11(a) without the beam size effect \((k_h = 1)\). To compare the proposed model to the RILEM, the following relationships \( \mu = 0.5(\sigma_2 + \sigma_3)/\sigma_{ct} \) and \( \sigma_3 = 0.85 \sigma_2 \) were assumed, which results \( 0.45 f_{Rm,1} = 1.18 \mu \sigma_{cr} \). \( A_{ct} \) is the area of concrete within tensile zone just before the crack occurs. Thus, \( A_{ct} = bh(1-k_1) \), where \( h = d/\alpha \) and \( k_1 \) is the expression for neutral axis when compressive and tensile stresses are in elastic stage, which is given in the Table 6.6. The last term, \( f_s \) is the maximum stress permitted in the reinforcement immediately after formation of crack (MPa). This parameter is taken equal to the yield strength \( f_{sy} \) in this study. For convenience in comparison, Eq. (6.13) was substituted by the relationships and assumptions mentioned in this paragraph and presented as
\[
\rho_{\text{min} 4} = \left( k_c k_e \sigma_{cr} - 1.18 \mu \sigma_{cr} \right) \frac{1-k_1}{\alpha f_{sy}}
\]  

(6.32)

Additional assumptions used in the model comparisons were: the cube concrete strength \( f_{ck} = 1.25 f_{c'} \); the effective depth \( d = 0.85 h \) or \( \alpha = 0.85 \); the concrete plastic stress \( \sigma_{cy} = 0.85 f_{c'} \); tensile strength of concrete in MPa was estimated by \( \sigma_{cr} = 0.623 f_{c}^{0.5} \), which yields a relatively high tensile strength to be used in the calculation of the minimum reinforcement ratio. Thus, the computed reinforcement ratio would fall on a conservative side. Concrete modulus of elasticity is assumed to be the same for tension and compression and estimated by \( E = E_c = 4735 f_{c}^{0.5} \). Two grades of steel \( f_{sy} = 275 \) MPa and 415 MPa were used in the comparisons. Postpeak tensile strength was set to zero (\( \mu = 0 \)) for conventional reinforced concrete system and one level of \( \mu = 0.2 \) for the hybrid system of FRC and steel rebar.

Fig. 6.12(a) compares the minimum reinforcement ratio for a conventional reinforced concrete system. It can be seen that all models require more reinforcement as the concrete strength increase. This is expected because all models estimate the tensile strength based on the relationship to the compressive stretch \( f_{c'} \). Among the four models compared, ACI suggests the most conservative value; followed by Shehata et al., the proposed model and the RILEM model. Since Shehata’s model can capture the beam size effects, two depths of 200 mm and 1000 mm are presented. The prediction reveals that the deeper sections require less reinforcement than the shallower ones. This trend agrees with the conclusions and experimental results conducted by Bosco et al. (1989&1990). Two depths of 200 mm and 1000 mm were also used in the RILEM model as the parameter \( k_e \) is sensitive to the
depth. It is interesting to note that the parameter $k_e$ accounting for nonuniform self-equilibrating stress produce the results similar to the beam size effect that the deeper section (1000 mm) requires less reinforcement. In contrast, Rao et al. (2007) derived an analytical model using crack mount opening displacement (CMOD) and stabilized crack growth as a criterion to establish the minimum flexural reinforcement in RC beams. The model yielded an opposite conclusion that the increase in beam depth causes more brittleness and requires higher flexural reinforcement to have stable crack propagation. When the reinforcement ratios determined by the four models (not include Rao et al. 2007) are compared to the experimental data as marked by the circle symbols, they are fairly agreed with the test data, with the ACI approach being the most conservative. The proposed model predicts the range of values between the less conservative RILEM and the more conservative Shehata et al.’s.

In the hybrid system that steel rebar was used with FRC instead of plain concrete, only the proposed and RILEM models are applicable in the determination of the minimum reinforcement. Therefore only two models were used in the comparisons at a fixed beam depth of 500 mm. As shown in Fig. 6.12(b), both models allow the minimum flexural reinforcement to be reduced when FRC matrix has postpeak tensile strength. Note that the parameter $\mu$ of 0.2 used in the proposed method has the same postpeak strength to the term $f_{Rm,i}/\sigma_{cr}$ of 0.52 used in the RILEM model as the assumption explained before ($0.45f_{Rm,i} = 1.18\mu\sigma_{cr}$). The proposed method suggests more conservative values than the RILEM does. Without test data for the FRC with rebar system, it cannot be identified which model give more reasonable results.
6.8. Algorithm to Predict Load-Deflection Responses

Load-deflection response of four point bending test can be simulated in the same way as the algorithm described in the previous chapter, which uses moment-curvature diagram, crack localization rules and moment-area method. The steps are summarized as follows.

For a given set of material parameters and beam dimensions, a moment curvature diagram can be generated by substitution of material and geometrical parameters to the closed form solutions presented in Table 6.6 and Eqs. (6.9) and (6.10), (6.13) and (6.14).

The applied load vector $P$ for simulation of beam test can be obtained from the discrete points along the moment curvature diagram as $P = 2M/X$ where $X$ is the distance from the support to the first point load.

The beam is segmented into finite sections. For a given load step, moment distribution along the beam can be determined by static equilibrium and the curvatures at discrete sections are identified by moment curvature relationship with crack localization rules.

The deflection at mid-span is then calculated by moment-area method of discretized curvatures between the support and mid-span. The procedure is repeated for each load step until a complete load-deflection response is obtained.

6.9. Experimental Verifications of Flexural Model

Full scale beam tests from the Brite Eiram project BRPR-CT98-0813 “Test and design methods for steel fibre reinforced concrete” were used in the model verification (Dupont 2003). The experimental program studied the effects of four variables: concrete strengths, fiber dosages, span lengths and longitudinal reinforcement ratios. Table 6.8 provides the details of the 12 beam series, each had 2 replicates. The specimens were made of two
grades of concrete: normal strength concrete (NSC) and high strength concrete (HSC). Normal strength concrete used fiber type RC 65/60 BN at 25 kg/m$^3$ and 50 kg/m$^3$ while HSC used fiber type RC 80/60 BP at 60 kg/m$^3$. All beams had the same cross section of 0.20 x 0.20 m but two different span lengths of 1.0 and 2.0 m. All beams were tested under four point bending set up in which the spacing between the two point loads was kept constant at 0.2 m. The first half of the series (B1 - B6) is SFRC beams, containing no rebar and the other half (B6 - B12) is hybrid reinforced concrete (HRC) beams, using SFRC and two rebar of size 8, 12 and 16 mm. Only one grade of steel was used, which had Young’s modulus of 200 GPa and yield stress of 560 MPa. The concrete cover to the reinforcement at the bottom of all beams was 15 mm.

The algorithm described in the previous section was used to simulate load-deflection responses of the 12 beam series. For assessment of the proposed method, load-deflection responses predicted by the Updated RILEM method (Dupont 2003) are also plotted as references. For a fair comparison of the two methods, the material models used in the proposed method were derived from the models used in the original publication (Dupont 2003). This was done by fitting the tension and compression models shown in Figs. 6.1(a and b) to the models shown in Figs. 6.11(a and b) such that the area under curves of the models used in both methods are the same. The original concrete parameters for the RILEM are presented in Table 6.9 and the obtained equivalent parameters for the proposed models are presented in Table 6.10. It is noted that the steel models of the two methods shown in Fig. 6.1(c) and Fig. 6.11(c) are exactly the same; thus, curve fitting was not required.
Fig. 6.13 shows the SFRC models for three levels of fiber contents (25 kg/m$^3$, 50 kg/m$^3$, and 75 kg/m$^3$) used in the proposed and Updated RILEM methods in the simulations of the first 6 SFRC beams (B1 - B6). Similarly, Fig. 6.14 shows the SFRC models used in the two methods for the HRC beams (B7-B12). The models presented in Fig. 6.13 and 6.14 are slightly different due to minor variability in the materials. The constant postpeak tensile stress used in the proposed tension model in Figs. 6.13 and 6.14 corresponds to an average postpeak response of the RILEM method. Thus, the areas under curves of the two tension models are the same. The ultimate tensile strains for both models are terminated at 0.025, as suggested by the RILEM method (Vandewalle et al. 2003). Figs. 6.13 and 6.14 also reveal that the compression model used in the proposed method is less stiff initially but stiffer than the RILEM’s in the region close to the ultimate strength; as a result, the areas under curves of both models are the same. Finally, the same elastic-perfectly plastic steel model for steel grade 560 MPa was used for the rebar in the simulations by the two methods: the proposed and RILEM.

Fig. 6.15 shows the simulations of the first 6 SFRC beams (without flexural reinforcement) representing the effect of concrete strength, fiber content, and span length. The experimental response of each beam series is obtained by averaging the test results of two replicates and it is compared to the simulation curves using the material models presented in Fig. 6.13. In general, the proposed method predicts load-deflection response slightly lower than the RILEM’s. This is due to the proposed method underestimates the descending postcrack response in the tension model and the initial compressive response in the compression model (Fig. 6.13). As the fiber content increases, the effect of
descending part become smaller as compared to the overall increase in the postpeak tensile capacity. The corresponding compressive stress developed in beam section also increases and the compressive modulus at higher stress level dominates the initial modulus. Thus, the load-deflection responses predicted by the two methods become closer at high fiber contents as demonstrated in Fig. 6.15(c.1 and c.2). Overall, the simulations of both methods are comparable to the experimental results.

Fig. 6.16 presents load-deflection responses of the second 6 HRC beams (with flexural reinforcement). Compare to Fig. 6.15, it is noticeable that the responses predicted by the Updated RILEM and the proposed methods are close to one another and show better agreement to the experimental results than the first 6 SFRC beams. This can be explained by the fact that when the SFRC beams are reinforced with steel rebar, their behaviors are controlled by the more predictable steel rebar instead of tensile strength of SFRC. The difference between the predictability of the two methods almost diminishes since they both use the same elastic-perfectly plastic steel model.

6.10. Evaluation of Ultimate Strength

Table 6.11 shows the maximum experimental flexural loads with the simulation results. For comparison of ultimate load capacity, the ultimate moment at infinity \( M_\infty \) predicted by simplified Eq. (6.26) is converted to the corresponding ultimate load at infinity \( P_\infty \) by the geometry of the four point bending set up:

\[
P_\infty = \frac{4M_\infty}{L - S_{mid}}
\]  

(6.33)
where $S_{mid}$ was the spacing between the two point loads, which was 0.2 m in this test program and $L$ was the clear span (1.0 or 2.0 m).

For SFRC beams (B1 - B6), the Updated RILEM method overestimates the maximum experimental loads with an average prediction/experiment ratio of $1.041 \pm 0.153$ whereas the proposed method underpredicts the results with $0.938 \pm 0.182$ as shown in Table 6.11. Note that the number after “±” represents the standard deviation. In the last column, the use of simplified Eq. (6.26) to calculate $P_\infty$ by Eq. (6.33) results the average prediction/experiment ratio of $1.044 \pm 0.22$, implying that the simplified Eq. (6.26) can reasonably estimate the flexural capacity for a fraction of computation time required by the RILEM or the proposed methods. In HRC beams (B7 - B12), both methods show significant improvement in prediction of load capacity. The average prediction/experiment ratio of RILEM method is $1.019 \pm 0.039$ as compared to $1.028 \pm 0.047$ for the proposed method. The prediction results of using simplified equation $(1.086\pm 0.054)$ is less accurate in term of average value but more reliable in term standard deviation.

6.11. Reduction Factors

The reduction factor $\phi_r$ is used in the design procedures to decrease a nominal strength of structural member due to variability in materials and workmanships such that the probability of failure is less than an acceptable level. This factor is normally obtained by statistical analysis of larger numbers of beams tested, coving various parameters expected in practice. Nevertheless, the results of strength prediction ratio in the previous section were used to estimate the reduction factors $\phi_{r1}$ and $\phi_{r2}$ for using with the simplified
equation to predict flexural strength of SFRC and HRC beams. These two factors can be updated with the availability of comprehensive test data.

Let $W_i$ represents the prediction/experiment ratio of the SFRC system in the last column of Table 6.11. If $W_i$ is less than or equal to one, it means the member strength predicted by the simplified Eq. (6.26) is less than or equal to the actual strength from beam tests, which yields the safe design. The constant reduction factor $\phi_i$ is multiplied to $W_i$ to obtain a reduced prediction/experiment ratio $w_i$. The average $u$ and standard deviation $s$ of $w_i$ can be determined from the following relationships.

$$u = \phi_i U; \quad s = \phi_i S$$ (6.34)

Where $U$ and $S$ are the average and standard deviation of $W_i$, respectively. To determine reduction factor $\phi_{r1}$ for SFRC system (B1 – B6), substitute $U = 1.044$ and $S = 0.22$ obtained from the last column of Table 6.11 in Eq. (6.34), which yields $u = 1.044\phi_{r1}$ and $s = 0.22\phi_{r1}$. For a small samples size, the t-distribution is used instead of normal distribution to identify the reduction factor that makes $w_i$ less than or equal to one, which can be expressed as

$$u + t_{p,q-1} \frac{s}{\sqrt{q}} \leq 1$$ (6.35)

where $p$ is the probability of error in testing hypothesis and $q-1$ is the degree of freedom calculated from sample size $q$ of 6. The t-distribution factor for a confidence level of 99%, $t_{0.01,5}$ is 3.365 (Montgomery 2001). By substituting $u$, $s$, $q$ and $t_{0.01,5}$ in Eq.(6.35), the $\phi_{r1}$ of 0.743 is obtained. Similarly, the reduction factor $\phi_{r2}$ for HRC beams (B7 - B12) can be obtained by the same procedure using $U = 1.086$ and $S = 0.054$ as presented at the
bottom of the last column of Table 6.11. The reduction factor \( \phi_r \) of 0.862 is obtained, which is close to 0.90, the reduction factor for nominal moment capacity of reinforced concrete beam specified by ACI-318 (2005). As expected, the reduction factor \( \phi_r \) is less than \( \phi_r \) because the behavior of SFRC beams is less predictable than HRC beams.

### 6.12. Chapter Conclusions

Material characteristics of FRC are idealized using elastic-constant postpeak strength for tension and elastic-perfectly plastic for compression. A steel model is approximated by an elastic-perfectly plastic relationship. The final closed form solutions for neutral axis depth ratio, moment curvature response and effective stiffness at each flexural deformation were expressed in normalized forms with respect to their values at cracking of plain FRC.

The parametric studies demonstrate that the use of discrete fibers to increase postpeak tensile strength is not as effective as continuous reinforcement in improving the moment capacity. Nevertheless, the ability of the fibers to distribute cracking potential leads to higher post crack strength than plain concrete. A hybrid system using FRC and ordinary reinforcement such as rebars or wire mesh provides an alternative cost saving solution for structural members subjected to moderate bending stresses.

Equations derived in this chapter paper can be used to generate the normalized design chart, representing the normalized ultimate moment capacity as a function of postpeak tensile strength and reinforcement ratio, which is applicable for three composite systems: conventional reinforced concrete, fiber reinforced concrete and hybrid reinforced concrete. The equation for determining normalized ultimate moment can be converted to
the more familiar coefficient of resistance $R$ and the results shows that it agrees well with the $R$ calculated by ACI stress block approach.

Two simplified equations are proposed for determination of the moment capacity at ultimate stage and the minimum reinforcement ratio to prevent a sudden failure after flexural cracking in serviceability stage. Numerical tests covering practical materials and geometrical ranges confirmed that the proposed equations compared well with their original equations.

The simulations of load-deflection responses of SFRC beams varying in concrete strengths, fiber contents, span lengths, and reinforcement ratios by the proposed and Updated RILEM methods were compared. Three observations can be made for the proposed model as compared to the RILEM approach. First, the RILEM method predicts higher load-deflection response than the proposed method for SFRC beams, especially at low fiber content specimens; however, the predictions by the two methods at higher fiber contents become close to one another. This can be explained by the fact that flexural behavior is less sensitive to the variations in flaw distribution in the concrete materials that have high postpeak tensile strength. At low fiber dosage, the equivalent tension model used in the proposed method underestimates the descending postpeak tensile response; thus, it predicts lower ultimate load. At higher fiber dosages, the descending part becomes negligible compared to the whole postpeak tensile response. Thus, the equivalent tension model gives a good estimation of the original model and the two methods yield approximately the same results. Secondly, the predicted responses by the two methods are approximately the same in HRC beams. This is due to the flexural
behavior of a HRC beam being more controlled by the flexural reinforcement than the tensile strength of SFRC. Since the proposed and Updated RILEM use the same elastic-perfectly plastic steel model, the predictions of the two methods are about the same. Finally, both methods show better prediction for the HRC beams than SFRC beams. The improved predictability due to the failure of HRC beams is governed by the yielding of steel, which has less variation than the tensile strength of SFRC. Thus, the steel model used in both methods can accurately predict the flexural responses.

The simplified design equation for predicting ultimate moment capacity was evaluated with the experiments. It was found that the equation slightly overestimates the flexural capacity of the SFRC beams and moderately overestimates HRC beams. Reduction factors based on small sample size for each case was determined, $\phi_1 = 0.743$ for SFRC system and $\phi_2 = 0.862$ for HRC system. These two numbers implies that the SFRC beams are less predictable than the HRC beams and need more reduction in the moment capacity predicted by the simplified equation.

**Notation**

*The following symbols are used in this chapter:*

- $A_s$ = area of steel reinforcement;
- $B_{1.5}$ = coefficients for neutral axis depth ratio in Table 6.6;
- $b$ = beam width;
- $C_{1.11}$ = coefficients for normalized moment in Table 6.6;
- $d$ = effective depth at location of steel reinforcement;
- $E$ = elastic tensile modulus of concrete;
Ec = elastic compressive modulus of concrete;

Es = elastic modulus of steel;

F = force components in stress diagram;

f = stress;

f\text{'}_c = cylindrical ultimate compressive strength of concrete;

f_{ck} = cube compressive strength of concrete;

f_{e,3} = equivalent flexural strength at deflection of 3 mm;

f_{fct,ef} = tensile strength of the concrete effective at the time when the cracks first form;

f_{Rm,1} = average residual flexural tensile strength;

G_1, G_2 = coefficients for calculation of the minimum flexural reinforcement;

h = full depth of a beam section;

K = effective flexural stiffness of beam section;

k = neutral axis depth ratio;

k_c = a coefficient taking account the form of the loading;

k_e = a coefficient which allows for the effect of non-uniform self equilibrating stress;

k_p = a coefficient taking account of the prestressing effect;

L = a clear span of beam;

M = moment;

M_n = nominal moment capacity;

M_u = ultimate moment;
\( n \) = modulus ratio \((E_s/E)\);
\( P \) = applied load vector in simulation of bending test;
\( P_{\text{max}} \) = the maximum load of bending test;
\( P_\infty \) = the ultimate load corresponding to the moment at infinity \( M_\infty \);
\( p \) = the probability of error in testing hypothesis;
\( q \) = degree of freedom;
\( R \) = coefficient of resistance;
\( S \) = standard deviation of \( W_i \);
\( S_{\text{mid}} \) = the spacing between the two point loads in four-point bending test;
\( s \) = standard deviation of \( w_i \);
\( t_{p,q-1} \) = statistical \( t \)-distribution factor
\( U \) = average value of \( W_i \);
\( u \) = mean value of \( w \);
\( W_f \) = fiber content \((\text{kg/m}^3)\);
\( W_i \) = the ratio of predicted ultimate load to the maximum experimental load;
\( w_i \) = the ratio of reduced predicted ultimate load to the maximum experimental load;
\( X \) = the distance from the support to the first point load;
\( y \) = moment arm from force component to neutral axis;
\( \alpha \) = normalized depth of steel reinforcement \((d/h)\);
\( \beta \) = normalized tensile strain at bottom fiber \((\varepsilon_{\text{bot}}/\varepsilon_{\text{cr}})\);
\( \beta_1 \) = coefficient for the plastic depth in ACI rectangular stress block;
ε = strain;

ε_c = concrete compressive strain;

ε_c0 = concrete compressive strain peak stress;

ε_c-top = concrete compressive strain at top fiber;

ε_t = concrete tensile strain;

ε_bot = concrete tensile strain at bottom fiber;

φ = curvature;

φ_{11}, φ_{2} = reduction factor for SFRC and hybrid SFRC-rebar beams, respectively;

γ = normalized concrete compressive modulus (E_c/E);

κ = normalized steel yield strain (ε_{sy}/ε_{cr});

κ_h = size dependent safety factor;

λ = normalized compressive strain at top fiber (ε_{c-top}/ε_{cr});

λ_{R1} = normalized compressive strain at top fiber when tensile strain at bottom fiber reaches cracking strain (end of region 1);

μ = normalized postpeak tensile strength (σ_p/σ_{cr});

μ_{crit} = the critical normalized postpeak tensile strength that changes deflection-softening to deflection-hardening;

Ω = mechanical reinforcement ratio (ρ_{fy}/f_{c'});

ρ = reinforcement ratio per effective area (A_s/bd);

ρ_g = reinforcement ratio per gross area (A_s/bh);
\( \rho_{bal} \) = reinforcement ratio at balance failure of beam using effective section \((bd)\);

\( \rho_{g, bal} \) = reinforcement ratio at balance failure of beam using gross section \((bh)\);

\( \rho_{\text{min}} \) = minimum flexural reinforcement ratio per effective area \((A_s/bd)\);

\( \rho_{\text{min}1}, \rho_{\text{min}2}, \rho_{\text{min}3}, \rho_{\text{min}4} \) = minimum reinforcement per effective area \((bd)\) suggested by ACI code, Sheheta et al., Proposed model and RILEM, respectively;

\( \sigma \) = concrete stress;

\( \sigma_c \) = concrete compressive stress;

\( \sigma_1, \sigma_2, \sigma_3 \) = the first, second and third stress points defined in the RILEM tensile stress strain model;

\( \sigma_p \) = constant postpeak tensile strength;

\( \sigma_t \) = concrete tensile stress; and

\( \omega \) = normalized concrete compressive yield strain \((\varepsilon_{cy}/\varepsilon_{cr})\).

**Superscripts**

' = normalizing symbol

**Subscripts**

\( 1 \) = stage 1, elastic compression – elastic tension;

\( 21 \) = stage 2.1, elastic compression – postpeak tension, steel is elastic;

\( 22 \) = stage 2.2, elastic compression – postpeak tension, steel is yield;

\( 31 \) = stage 3.1, plastic compression – postpeak tension, steel is elastic;
32 = stage 3.2, plastic compression – postpeak tension, steel is yield;
c1 = elastic compression zone 1 in stress diagram;
c2 = plastic compression zone 2 in stress diagram;
cr = at cracking;
cu = at ultimate concrete compressive strain;
cy = at concrete compressive yielding;
i = at stage \( i \) of normalized concrete compressive strain and tensile steel condition;
s = refer to steel;
sy = at steel yielding;
t1 = elastic tension zone 1 in stress diagram;
t2 = postpeak tension zone 2 in stress diagram;
tu = at concrete ultimate tensile stain;
cu = at concrete ultimate compressive strain; and
\( \infty \) = at concrete compressive strain approach infinity.
Table 6.1. Normalized Height of Compression and Tension Zones for Each Stage of Normalized Compressive Strain at Top Fiber ($\lambda$)

<table>
<thead>
<tr>
<th>Normalized height</th>
<th>Stage 1 $0 &lt; \lambda \leq \lambda_{R1}$</th>
<th>Stage 2 $\lambda_{R1} &lt; \lambda \leq \omega$</th>
<th>Stage 3 $\omega &lt; \lambda \leq \lambda_{cu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2.1</td>
<td>2.2</td>
</tr>
<tr>
<td>$h_{c2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{h_{c2}}{h}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{c1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{h_{c1}}{h}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{h1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{h_{h1}}{h}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{h2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{h_{h2}}{h}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\epsilon_{s} < \epsilon_{sy}$ $\epsilon_{s} > \epsilon_{sy}$ $\epsilon_{s} \leq \epsilon_{sy}$ $\epsilon_{s} > \epsilon_{sy}$

$h_{c2} = - k \frac{\lambda}{\omega} h$

$h_{c1} = k \frac{\lambda}{\omega} h$

$h_{h1} = 1 - k \frac{\lambda}{\lambda} h$

$h_{h2} = - \frac{\lambda - (\lambda + 1)k}{\lambda} h$
Table 6.2. Normalized Stress at Vertices in the Stress Diagram for Each Stage of Normalized Compressive Strain at Top Fiber ($\lambda$)

<table>
<thead>
<tr>
<th>Normalized stress</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \lambda \leq \lambda_{R1}$</td>
<td>$\lambda_{R1} &lt; \lambda \leq \omega$</td>
<td>$\omega &lt; \lambda \leq \lambda_{cu}$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_s &lt; \varepsilon_{sy}$</td>
<td>$\varepsilon_s &gt; \varepsilon_{sy}$</td>
<td>$\varepsilon_s \leq \varepsilon_{sy}$</td>
<td>$\varepsilon_s &gt; \varepsilon_{sy}$</td>
</tr>
</tbody>
</table>

| $\frac{f_{c2}}{E\varepsilon_{cr}}$ | - | - | $\gamma \omega$ |
| $\frac{f_{c1}}{E\varepsilon_{cr}}$ | $\gamma \lambda$ | $\gamma \lambda$ | $\gamma \omega$ |
| $\frac{f_{i1}}{E\varepsilon_{cr}}$ | $\frac{1-k}{k} \lambda$ | 1 | 1 |
| $\frac{f_{i2}}{E\varepsilon_{cr}}$ | - | $\mu$ | $\mu$ |
| $\frac{f_s}{E\varepsilon_{cr}}$ | $\frac{n\lambda(a-k)}{k}$ | $\frac{n\lambda(a-k)}{k}$ | $n\kappa$ | $\frac{n\lambda(a-k)}{k}$ | $n\kappa$ |
Table 6.3. Normalized Force Component for Each Stage of Normalized Compressive Strain at Top Fiber ($\lambda$)

<table>
<thead>
<tr>
<th>Normalized force component</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \lambda \leq \lambda_{R1}$</td>
<td>$\lambda_{R1} &lt; \lambda \leq \omega$</td>
<td>$\omega &lt; \lambda \leq \lambda_{cu}$</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>2.2</td>
<td>3.1</td>
<td>3.2</td>
</tr>
<tr>
<td>$\varepsilon_s \leq \varepsilon_{sy}$</td>
<td>$\varepsilon_s &gt; \varepsilon_{sy}$</td>
<td>$\varepsilon_s \leq \varepsilon_{sy}$</td>
<td>$\varepsilon_s &gt; \varepsilon_{sy}$</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{F_{e2}}{bhE\varepsilon_{cr}} &= -
\gamma \omega k
\frac{(\lambda - \omega)}{\lambda} \\
\frac{F_{e1}}{bhE\varepsilon_{cr}} &= \frac{1}{2} \gamma \lambda k
\frac{1}{2} \gamma \lambda k
\gamma \omega^2 k \\
\frac{F_{11}}{bhE\varepsilon_{cr}} &= \frac{\lambda}{2k} (k-1)^2
\frac{k}{2\lambda}
\frac{k}{2\lambda} \\
\frac{F_{12}}{bhE\varepsilon_{cr}} &= -
\frac{\mu}{\lambda} (\lambda - k\lambda - k)
\frac{\mu}{\lambda} (\lambda - k\lambda - k) \\
\frac{F_s}{bhE\varepsilon_{cr}} &= \frac{\rho_{g} n \lambda}{k} (\alpha - k)
\frac{\rho_{g} n \lambda}{k} (\alpha - k)
\frac{\rho_{g} n \kappa}{k} (\alpha - k)
\frac{\rho_{g} n \kappa}{k} (\alpha - k) \\
\end{align*}
\]
Table 6.4. Normalized Moment Arm of Force Component for Each Stage of Normalized Compressive Strain at Top Fiber (\(\lambda\))

<table>
<thead>
<tr>
<th>Normalized moment arm</th>
<th>Stage 1</th>
<th>Stage 2.1</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 &lt; \lambda \leq \lambda_{RI})</td>
<td>(\lambda_{RI} &lt; \lambda \leq \omega)</td>
<td>(\omega &lt; \lambda \leq \lambda_{cu})</td>
<td>(\varepsilon_s \leq \varepsilon_{sy})</td>
</tr>
<tr>
<td>(2.1)</td>
<td>(2.2)</td>
<td>(3.1)</td>
<td>(3.2)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{y_{c2}}{h} & = - \quad - \quad \frac{k(\omega + \lambda)}{2\lambda} \\
\frac{y_{c1}}{h} & = \frac{2k}{3} \quad \frac{2k}{3} \quad \frac{2\omega k}{3\lambda} \\
\frac{y_{t1}}{h} & = \frac{2}{3}(1-k) \quad \frac{2k}{3\lambda} \quad \frac{2k}{3\lambda} \\
\frac{y_{t2}}{h} & = - \quad \frac{\lambda + (1 - \lambda)k}{2\lambda} \quad \frac{\lambda + (1 - \lambda)k}{2\lambda} \\
\frac{y_{s}}{h} & = \alpha - k \quad \alpha - k \quad \alpha - k
\end{align*}
\]
Table 6.5. Equilibrium of Force, Moment, Curvature and Stiffness for Each Stage of Normalized Compressive Strain at Top Fiber ($\lambda$)

<table>
<thead>
<tr>
<th>Stage</th>
<th>Net Section Force, $\Sigma F$</th>
<th>Moment, $M$</th>
<th>Curvature, $\phi_1$</th>
<th>Stiffness, $K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-F_{cl} + F_{t_1} + F_s$</td>
<td>$F_{cl}y_{cl} + F_{t_1}y_{t_1} + F_sy_s$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>$-F_{cl} + F_{t_1} + F_{t_2} + F_s$</td>
<td>$F_{cl}y_{cl} + F_{t_1}y_{t_1} + F_{t_2}y_{t_2} + F_sy_s$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>$-F_{cl} + F_{t_1} + F_{t_2} + F_s$</td>
<td>$F_{cl}y_{cl} + F_{t_1}y_{t_1} + F_{t_2}y_{t_2} + F_sy_s$</td>
<td>$\varepsilon_{ctop}$</td>
<td>$\frac{M}{\phi}$</td>
</tr>
<tr>
<td>3.1</td>
<td>$-F_{cl} - F_{c_2} + F_{t_1} + F_{t_2} + F_s$</td>
<td>$F_{cl}y_{cl} + F_{c_2}y_{c_2} + F_{t_1}y_{t_1} + F_{t_2}y_{t_2} + F_sy_s$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.2</td>
<td>$-F_{cl} - F_{c_2} + F_{t_1} + F_{t_2} + F_s$</td>
<td>$F_{cl}y_{cl} + F_{c_2}y_{c_2} + F_{t_1}y_{t_1} + F_{t_2}y_{t_2} + F_sy_s$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 6.6. Normalized Neutral Axis, Moment, Curvature and Stiffness for Each Stage of Normalized Compressive Strain (\(\lambda\))

<table>
<thead>
<tr>
<th>Stage</th>
<th>(k)</th>
<th>(M')</th>
<th>(\phi')</th>
<th>(K')</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(k_1 = \frac{1 + \rho_g n - \sqrt{\rho_g^2 n^2 + 2 \rho_g n (1 - \alpha + \alpha \gamma)} + \gamma}{1 - \gamma}) for (\gamma \neq 1)</td>
<td>(M'_1 = \frac{2 \lambda}{k_1} \left[ C_1 k_1^3 + C_2 k_1^2 + C_3 k_1 + C_4 \right])</td>
<td>(\phi'_1 = \frac{\lambda}{2 k_1})</td>
<td>(K'_1 = \frac{M'_1}{\phi'_1})</td>
</tr>
<tr>
<td>2.1</td>
<td>(k_{21} = \frac{\lambda}{B_1} \left( B_2 + \sqrt{B_3 + 2 \alpha \rho_g n B_1} \right))</td>
<td>(M'<em>{21} = \frac{C_5 k</em>{21}^3 + C_6 k_{21}^2 + C_7 k_{21} + C_8}{\lambda^2 k_{21}})</td>
<td>(\phi'<em>{21} = \frac{\lambda}{2 k</em>{21}})</td>
<td>(K'<em>{21} = \frac{M'</em>{21}}{\phi'_{21}})</td>
</tr>
<tr>
<td>2.2</td>
<td>(k_{22} = \frac{B_4}{B_1})</td>
<td>(M'<em>{22} = \frac{C_5 k</em>{22}^3 + C_9 k_{22} + C_{10}}{\lambda^2 k_{22}})</td>
<td>(\phi'<em>{22} = \frac{\lambda}{2 k</em>{22}})</td>
<td>(K'<em>{22} = \frac{M'</em>{22}}{\phi'_{22}})</td>
</tr>
<tr>
<td>3.1</td>
<td>(k_{31} = \frac{\lambda}{B_5} \left( B_2 + \sqrt{B_3 + 2 \alpha \rho_g n B_5} \right))</td>
<td>(M'<em>{31} = \frac{C</em>{11} k_{31}^3 + C_6 k_{31}^2 + C_7 k_{31} + C_8}{\lambda^2 k_{31}})</td>
<td>(\phi'<em>{31} = \frac{\lambda}{2 k</em>{31}})</td>
<td>(K'<em>{31} = \frac{M'</em>{31}}{\phi'_{31}})</td>
</tr>
<tr>
<td>3.2</td>
<td>(k_{32} = \frac{B_4}{B_5})</td>
<td>(M'<em>{32} = \frac{C</em>{11} k_{32}^3 + C_9 k_{32} + C_{10}}{\lambda^2})</td>
<td>(\phi'<em>{32} = \frac{\lambda}{2 k</em>{32}})</td>
<td>(K'<em>{32} = \frac{M'</em>{32}}{\phi'_{32}})</td>
</tr>
</tbody>
</table>

where

\[
B_1 = \gamma \lambda^2 + 2 \mu (\lambda + 1) - 1; \quad B_2 = \mu - \rho_g n \lambda; \quad B_3 = \rho_g n \left( \rho_g n \lambda^2 - 2 \mu \lambda \right) + \mu^2; \quad B_4 = 2 \lambda \left( \rho_g n \kappa + \mu \right); \quad B_5 = 2 \gamma \omega \lambda - \gamma \omega^2 + 2 \mu (\lambda + 1) - 1; \quad C_1 = \gamma - 1;
\]

\[
C_2 = 3 \left( \rho_g n + 1 \right); \quad C_3 = -3 \left( 2 \rho_g n \alpha + 1 \right); \quad C_4 = 3 \rho_g n \alpha^2 + 1; \quad C_5 = 2 \gamma \lambda^3 + 3 \mu (\lambda^2 - 1) + 2; \quad C_6 = 6 \lambda^2 \left( \rho_g n \lambda - \mu \right); \quad C_7 = 3 \lambda^2 \left( \mu - 4 \rho_g n \alpha \lambda \right);
\]

\[
C_8 = 6 \rho_g n \alpha^2 \lambda; \quad C_9 = -6 \lambda^2 \left( \rho_g n \kappa + \mu \right); \quad C_{10} = 3 \lambda^2 \left( 2 \rho_g n \kappa + \mu \right); \quad C_{11} = 3 \gamma \omega \lambda^2 - \gamma \omega^3 + 3 \mu (\lambda^2 - 1) + 2
\]
Table 6.7. Parameters for Material models and Steel Rebar Locations for Using in Verification of the Simplified Equation for Ultimate Moment and Minimum Reinforcement Ratio

<table>
<thead>
<tr>
<th>$f_c'$</th>
<th>$E$</th>
<th>$\sigma_{cr}$</th>
<th>$\varepsilon_{cr}$</th>
<th>$\mu$</th>
<th>$\sigma_{cy}$</th>
<th>$\gamma$</th>
<th>$\omega$</th>
<th>$\varepsilon_{cu}$</th>
<th>$\lambda_{cu}$</th>
<th>$E_s$</th>
<th>$n$</th>
<th>$f_{sy}$</th>
<th>$\kappa$</th>
<th>$\alpha$</th>
</tr>
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<tbody>
<tr>
<td>MPa</td>
<td>MPa</td>
<td>MPA</td>
<td>psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.25 $f'_c$</td>
<td>0.00</td>
<td>1.0 $f'_c$</td>
<td>0.67</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>43</td>
<td>$\sqrt[4]{f'_c}$</td>
<td>0.63 $\frac{f'_c}{E}$</td>
<td>0.33</td>
<td>0.85 $f'_c$</td>
<td>1.00</td>
<td>$\frac{\sigma_{cy}}{\gamma E \varepsilon_{cr}}$</td>
<td>0.004</td>
<td>$\frac{\varepsilon_{cu}}{\varepsilon_{cr}}$</td>
<td>200</td>
<td>$\frac{E_s}{E}$</td>
<td>415</td>
<td>$\frac{f_{sy}}{E_s E \varepsilon_{cr}}$</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>$1.0 \sqrt[4]{f'_c}$</td>
<td>0.67</td>
<td>0.7 $f'_c$</td>
<td>1.33</td>
<td>0.005</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
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</tbody>
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Table 6.8. Beam Test Series

<table>
<thead>
<tr>
<th>Beam</th>
<th>Mix</th>
<th>Fiber content</th>
<th>Span</th>
<th>Rebar</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>kg/m³</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>NSC</td>
<td>25</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>B2</td>
<td>NSC</td>
<td>25</td>
<td>2.0</td>
<td>-</td>
</tr>
<tr>
<td>B3</td>
<td>NSC</td>
<td>50</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>B4</td>
<td>NSC</td>
<td>50</td>
<td>2.0</td>
<td>-</td>
</tr>
<tr>
<td>B5</td>
<td>HSC</td>
<td>60</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>B6</td>
<td>HSC</td>
<td>60</td>
<td>2.0</td>
<td>-</td>
</tr>
<tr>
<td>B7</td>
<td>NSC</td>
<td>25</td>
<td>1.0</td>
<td>2-φ8</td>
</tr>
<tr>
<td>B8</td>
<td>NSC</td>
<td>25</td>
<td>2.0</td>
<td>2-φ8</td>
</tr>
<tr>
<td>B9</td>
<td>NSC</td>
<td>50</td>
<td>1.0</td>
<td>2-φ12</td>
</tr>
<tr>
<td>B10</td>
<td>NSC</td>
<td>50</td>
<td>2.0</td>
<td>2-φ12</td>
</tr>
<tr>
<td>B11</td>
<td>HSC</td>
<td>60</td>
<td>1.0</td>
<td>2-φ16</td>
</tr>
<tr>
<td>B12</td>
<td>HSC</td>
<td>60</td>
<td>2.0</td>
<td>2-φ16</td>
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</table>
Table 6.9. Steel Fiber Reinforced Concrete Parameters for Updated RILEM Model

<table>
<thead>
<tr>
<th>Beam Type</th>
<th>Mix</th>
<th>Fiber content kg/m³</th>
<th>$E$ (MPa)</th>
<th>$f_{c'}$ (MPa)</th>
<th>$\sigma_1$ (MPa)</th>
<th>$\sigma_2$ (MPa)</th>
<th>$\sigma_3$ (MPa)</th>
<th>$\varepsilon_1$ (%)</th>
<th>$\varepsilon_2$ (%)</th>
<th>$\varepsilon_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSC 25</td>
<td>31,854</td>
<td>30.2</td>
<td>3.5</td>
<td>1.1</td>
<td>0.8</td>
<td>0.011</td>
<td>0.21</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSC 50</td>
<td>30,564</td>
<td>26.6</td>
<td>4.2</td>
<td>2.0</td>
<td>1.2</td>
<td>0.014</td>
<td>0.24</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HSC 60</td>
<td>38,411</td>
<td>52.9</td>
<td>6.2</td>
<td>3.1</td>
<td>3.1</td>
<td>0.016</td>
<td>0.26</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSC 25</td>
<td>30,472</td>
<td>26.4</td>
<td>3.2</td>
<td>1.3</td>
<td>0.9</td>
<td>0.011</td>
<td>0.21</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HC 50</td>
<td>30,348</td>
<td>26.1</td>
<td>3.8</td>
<td>1.8</td>
<td>1.1</td>
<td>0.013</td>
<td>0.23</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HSC 60</td>
<td>39,002</td>
<td>55.4</td>
<td>6.3</td>
<td>3.8</td>
<td>3.2</td>
<td>0.016</td>
<td>0.26</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that strain at peak stress, $\varepsilon_{c0} = 0.2\%$ and ultimate compressive strain, $\varepsilon_{cu} = 0.35\%$ for all mixes.
Table 6.10. The Equivalent Steel Fiber Reinforced Concrete Parameters for the Proposed Model

<table>
<thead>
<tr>
<th>Beam Type</th>
<th>Mix</th>
<th>Fiber content</th>
<th>$E$</th>
<th>$E_c$</th>
<th>$\sigma_{cy}$</th>
<th>$\sigma_{cr}$</th>
<th>$\sigma_p$</th>
<th>$\varepsilon_{cr}$</th>
<th>$\varepsilon_{tu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSC</td>
<td>25</td>
<td>31,818</td>
<td>22,626</td>
<td>30.2</td>
<td>3.5</td>
<td>1.1</td>
<td>0.011</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>SFRC</td>
<td>NSC</td>
<td>50</td>
<td>30,000</td>
<td>19,985</td>
<td>26.6</td>
<td>4.2</td>
<td>2.0</td>
<td>0.014</td>
<td>25</td>
</tr>
<tr>
<td>HSC</td>
<td>60</td>
<td>38,750</td>
<td>39,670</td>
<td>52.9</td>
<td>6.2</td>
<td>3.1</td>
<td>0.016</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>NSC</td>
<td>25</td>
<td>29,091</td>
<td>19,805</td>
<td>26.4</td>
<td>3.2</td>
<td>1.3</td>
<td>0.011</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>HRC</td>
<td>NSC</td>
<td>50</td>
<td>29,231</td>
<td>19,565</td>
<td>26.1</td>
<td>3.8</td>
<td>1.8</td>
<td>0.013</td>
<td>25</td>
</tr>
<tr>
<td>HSC</td>
<td>60</td>
<td>39,375</td>
<td>41,530</td>
<td>55.4</td>
<td>6.3</td>
<td>3.8</td>
<td>0.016</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

Note that strain at compressive yield stress, $\varepsilon_{cy} = 0.133\%$ and ultimate compressive strain, $\varepsilon_{cu} = 0.35\%$ for all mixes
Table 6.11. Comparison of Ultimate Load Capacity

<table>
<thead>
<tr>
<th>Beam</th>
<th>$W_f$</th>
<th>Span</th>
<th>Rebar</th>
<th>$P_{max}^{Exp.}$</th>
<th>$P_{max}^{RILEM}$</th>
<th>$P_{max}^{Proposed}$</th>
<th>$P_{\infty}^{Eq.(6.33)}$</th>
<th>$P_{max}^{Exp.}/RILEM$</th>
<th>$P_{max}^{Proposed}/RILEM$</th>
<th>$P_{max}^{Exp.}/Eq.(6.33)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kg/m$^3$</td>
<td>m</td>
<td>mm</td>
<td>kN</td>
<td>kN</td>
<td>kN</td>
<td>kN</td>
<td>RILEM/Exp.</td>
<td>Proposed/RILEM</td>
<td>Eq.(6.33)/Exp.</td>
</tr>
<tr>
<td>B1</td>
<td>25</td>
<td>1</td>
<td>0</td>
<td>26.7</td>
<td>26.8</td>
<td>21.4</td>
<td>21.4</td>
<td>1.00</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>B2</td>
<td>25</td>
<td>2</td>
<td>0</td>
<td>10.3</td>
<td>11.3</td>
<td>8.2</td>
<td>9.5</td>
<td>1.10</td>
<td>0.79</td>
<td>0.92</td>
</tr>
<tr>
<td>B3</td>
<td>50</td>
<td>1</td>
<td>0</td>
<td>27.1</td>
<td>33.9</td>
<td>30.2</td>
<td>37.7</td>
<td>1.25</td>
<td>1.11</td>
<td>1.39</td>
</tr>
<tr>
<td>B4</td>
<td>50</td>
<td>2</td>
<td>0</td>
<td>16.9</td>
<td>14.2</td>
<td>13.4</td>
<td>16.8</td>
<td>0.84</td>
<td>0.79</td>
<td>0.99</td>
</tr>
<tr>
<td>B5</td>
<td>60</td>
<td>1</td>
<td>0</td>
<td>63.4</td>
<td>57.4</td>
<td>58.4</td>
<td>59.2</td>
<td>0.91</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>B6</td>
<td>60</td>
<td>2</td>
<td>0</td>
<td>21.5</td>
<td>24.6</td>
<td>26.0</td>
<td>26.3</td>
<td>1.14</td>
<td>1.21</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td></td>
<td>1.041</td>
<td>0.938</td>
<td>1.044</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td></td>
<td></td>
<td>0.153</td>
<td>0.182</td>
<td>0.220</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>B7</td>
<td>25</td>
<td>1</td>
<td>2-$\phi$8</td>
<td>70.5</td>
<td>69.4</td>
<td>69.5</td>
<td>73.8</td>
<td>0.98</td>
<td>0.99</td>
<td>1.05</td>
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<tr>
<td>B8</td>
<td>25</td>
<td>2</td>
<td>2-$\phi$8</td>
<td>27.7</td>
<td>30.2</td>
<td>30.9</td>
<td>32.8</td>
<td>1.09</td>
<td>1.11</td>
<td>1.18</td>
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<tr>
<td>B9</td>
<td>50</td>
<td>1</td>
<td>2-$\phi$12</td>
<td>130.0</td>
<td>131.7</td>
<td>130.5</td>
<td>138.8</td>
<td>1.01</td>
<td>1.00</td>
<td>1.07</td>
</tr>
<tr>
<td>B10</td>
<td>50</td>
<td>2</td>
<td>2-$\phi$12</td>
<td>55.7</td>
<td>57.6</td>
<td>58.0</td>
<td>61.7</td>
<td>1.03</td>
<td>1.04</td>
<td>1.11</td>
</tr>
<tr>
<td>B11</td>
<td>60</td>
<td>1</td>
<td>2-$\phi$16</td>
<td>250.0</td>
<td>245.8</td>
<td>249.5</td>
<td>259.9</td>
<td>0.98</td>
<td>1.00</td>
<td>1.04</td>
</tr>
<tr>
<td>B12</td>
<td>60</td>
<td>2</td>
<td>2-$\phi$16</td>
<td>108.1</td>
<td>109.2</td>
<td>110.9</td>
<td>115.5</td>
<td>1.01</td>
<td>1.03</td>
<td>1.07</td>
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<td>Average</td>
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<td>Standard Deviation</td>
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</table>
Fig. 6.1. Material model for single reinforced concrete design: (a) compression model; (b) tension model; (c) steel model; (d) beam cross section
Fig. 6.2. Strain and stress diagram at three stages of applied compressive strain at top fiber ($\lambda$): (a) stage 1: ($0 < \lambda \leq \lambda_{R1}$) elastic compression-elastic tension; (b) stage 2: ($\lambda_{R1} < \lambda \leq \omega$) elastic compression – postpeak tension; (c) stage 3 ($\omega < \lambda \leq \lambda_{cu}$) plastic compression – postpeak tension
Fig. 6.3. Material model for typical steel fiber reinforced concrete and rebar used in parametric studies: (a) concrete model; (b) steel model
Normalized Top Compressive Strain, $\lambda$

Neutral Axis Depth Ratio, $k$

(a) $\rho_g=0.00$

(b) $\mu=0.00$

$\rho_g=0.01$

$\rho_g=0.02$

$\rho_g=0.03$

Normalized Top Compressive Strain, $\lambda$
Fig. 6.4. Parametric studies of neutral axis depth ratio for different levels of postpeak tensile strength parameter $\mu$ and reinforcement ratio $\rho_g$. 
Normalized Moment, $M'$

- Normalized Curvature, $\phi'$
- $\mu = 1.00$
- $\mu = 0.67$
- $\mu = 0.33$
- $\mu = 0.00$

- $\rho_g = 0.00$
- $\rho_g = 0.01$
- $\rho_g = 0.02$
- $\rho_g = 0.03$
Fig. 6.5. Parametric studies of normalized moment curvature diagram for different levels of postpeak tensile strength parameter $\mu$ and reinforcement ratio $\rho_g$. 

(C) $\mu=1.00$, $\mu=0.67$, $\mu=0.33$, $\mu=0.00$ 

(d) $\rho_g=0.03$, $\rho_g=0.02$, $\rho_g=0.01$, $\rho_g=0.00$
Fig. 6.6. Parametric studies of normalized stiffness degradation for different levels of postpeak tensile strength parameter $\mu$ and reinforcement ratio $\rho_g$.
Fig. 6.7. Design chart of normalized ultimate moment capacity for different levels of post-crack tensile strength $\mu$ and reinforcement ratio $\rho_g$. 

- $f'_c=30$ MPa, $f_{sy}=400$ MPa
- $\gamma=1.00$, $\omega=8.5$, $\alpha=0.80$
- $n=8.33$, $\kappa=16.0$, $\lambda_{cu}=28.0$
The diagram illustrates the relationship between the reinforcement ratio ($\rho = A_s/bd$) and the coefficient of resistance $R$ (MPa). The figure shows data points and lines for different concrete strengths ($f_{c}'$) at given reinforcement ratios ($\rho_{g,bal}$). The diagram includes the following details:

- $f_{c}' = 55$ MPa
- $f_{c}' = 43$ MPa
- $f_{c}' = 30$ MPa
- $f_{sy} = 275$ MPa

The diagram also includes a legend with symbols for ACI, 2005, and a line labeled "Proposed Model."
Fig. 6.8. Compare coefficient of resistance using ACI stress block method and the proposed equation: (a) for steel grade 275 MPa; (b) for steel grade 415 MPa
Fig. 6.9. Evaluation of the simplified equation for predicting normalized ultimate moment capacity: (a) between theoretical prediction and simplified equation; (b) reliability of the simplified equation
Fig. 6.10. Verification of minimum reinforcement ratio: (a) compare predicted normalized ultimate moment capacity at the minimum reinforcement; (b) the required minimum reinforcement for different level of postpeak tensile strength.
Fig. 6.11. Material stress strain model for Updated RILEM method: (a) tension model for SFRC; (b) compression model for SFRC; (c) steel model
Fig. 6.12. Comparison of minimum reinforcement ratio; (a) conventional reinforced concrete beam; (b) FRC with steel rebar system
RILEM Proposed Model

(a) NSC, Fiber content 25 kg/m³

(b) NSC, Fiber content 50 kg/m³
Fig. 6.13. SFRC models for three levels of fiber contents (25 kg/m$^3$, 50 kg/m$^3$ and 75 kg/m$^3$) used in SFRC beam series
Tensile Stress (MPa)

Tensile Strain (mm/mm)

Compressive Stress (MPa)

Compressive Strain (mm/mm)

(a)

RILEM Proposed Model

NSC, Fiber content 25 kg/m³, 2-φ8 mm

(b)

RILEM Proposed Model

NSC, Fiber content 50 kg/m³, 2-φ12 mm
Fig. 6.14. SFRC models for three levels of fiber contents (25 kg/m$^3$, 50 kg/m$^3$ and 75 kg/m$^3$) used in HRC beam series.
(a.1) NSC, Fiber content 25 kg/m³, Span 1 m

(a.2) NSC, Fiber content 25 kg/m³, Span 2 m
(b.1)

NSC,
Fiber content 50 kg/m³,
Span 1 m

Experiment (Avg.)
RILEM
Proposed Model

(b.2)

NSC,
Fiber content 50 kg/m³,
Span 2 m

Experiment (Avg.)
RILEM
Proposed Model
Fig. 6.15. Load-deflection responses of SFRC beams at three levels of fiber contents (25 kg/m$^3$, 50 kg/m$^3$ and 75 kg/m$^3$)
Experiment (Avg.)  RILEM  Proposed Model

(a.1)
NSC,  
Fiber content 25 kg/m$^3$,  
Span 1 m,  
2-φ8 mm

(a.2)
NSC,  
Fiber content 25 kg/m$^3$,  
Span 2 m,  
2-φ8 mm
(b.1) NSC, Fiber content 50 kg/m³, Span 1 m, 2-φ12 mm
- ∅ Experiment (Avg.)
- RILEM
- Proposed Model

(b.2) NSC, Fiber content 50 kg/m³, Span 2 m, 2-φ12 mm
- ∅ Experiment (Avg.)
- RILEM
- Proposed Model
Fig. 6.16. Load deflection responses of HRC beams at three levels of fiber contents (25 kg/m³, 50 kg/m³ and 75 kg/m³)
CHAPTER 7
DESIGN GUIDELINES FOR FIBER REINFORCED CONCRETE

7.1. Introduction

A wide range of fiber reinforced concrete (FRC) systems have been invented and named differently according to their compositions and manufacturing techniques; for examples, steel fiber reinforced concrete (SFRC), glass fiber reinforced concrete (GFRC), engineered cementitious composite (ECC), steel infiltrated mat concrete (SIMCON) and other synthetic fiber systems. The composites are classified into two categories: strain-hardening and strain-softening, according to their hardening and softening tensile responses observed in tension tests. In strain-softening category, additional two subclasses: deflection-hardening and deflection-softening are defined according to their hardening and softening deflection responses observed in bending tests.

This chapter presents the design methodology for strain-softening fiber reinforced concrete members, covering flexural strength, shear capacity and minimum postpeak tensile strength and minimum flexural reinforcement. The design procedures are primarily based on the simplified equations developed in Chapters 5 and 6, ACI 318 (2005) and RILEM TC 162-TDF (Vandewalle et al. 20003). Several empirical equations and critical values used in literatures and building codes are primarily for SFRC. Therefore, the guideline for strain-softening presented here is more suitable for SFRC. Other fibers that can demonstrate comparable mechanical properties to the SFRC can follow this guideline with engineering judgments. In addition the design guide for strain-hardening fiber reinforced concrete is presented, covering only flexural strength of
GFRC. Design examples of both classes are provided to illustrate the calculation steps using the proposed equations.

7.2. Strain-Softening Fiber Reinforced Concrete

When discrete fibers are added into concrete mix to create strain-softening FRC, they generally improve several mechanical properties of the FRC, depending on the volume fraction, bond quality and uniformity of fiber dispersion. It is possible to establish empirical relationships between mechanical properties and the input design variables. However, this approach limits the applicability of the equation to the range of designs variables used in the establishment of the relationship. Since fibers negligibly improve strength and modulus of FRC, it assumes that the formulas for predicting mechanical properties of plain concrete suggested by well establish building code ACI-318 are applicable for FRC. Only the postcrack tensile response of FRC that is significantly different from the plain concrete will be obtained by material testing, which will be presented next.

7.2.1. Material Models and Parameters

For the design purpose, complex material characteristics of strain-softening FRC are simplified to uniaxial compressive and tensile stress strain models as shown in Fig. 7.1(a and b). When conventional steel rebar is used in the FRC system, an additional steel model presented in Fig. 7.1(c) is used in a rectangular beam section as shown in Fig. 7.1(d). The design equations were derived base on dimensionless concept which makes the equations applicable for both SI and Imperial unit. In this approach, material parameters are defined as normalized parameters with respect to the cracking tensile
strain $\varepsilon_{cr}$ and Young’s modulus $E$. Geometric parameters are defined as normalized parameters with respect to beam width $b$ and depth $h$.

### 7.2.1.1. Tension Model

Fig. 7.1(a) shows the idealized tension model that stress increases linearly from zero to the cracking tensile strength $\sigma_{cr}$ at cracking tensile strain $\varepsilon_{cr}$. After cracking, stress drops to the postpeak strength level $\sigma_p$ and remains constant until it is terminated at the ultimate tensile strain $\varepsilon_{tu}$. Two normalized material parameters for the tension model are defined as follows

$$\mu = \frac{\sigma_p}{E \varepsilon_{cr}} = \frac{\sigma_p}{\sigma_{cr}} \quad (7.1)$$

$$\beta_{tu} = \frac{\varepsilon_{tu}}{\varepsilon_{cr}} \quad (7.2)$$

where $\mu$ is the normalized postpeak tensile strength and $\beta_{tu}$ is the normalized ultimate tensile strain. For SFRC system, the ultimate tensile strain is 0.025 according to RILEM model (Vandewalle et al. 2000, 2003). Cracking tensile strength and Young’s modulus in psi can be estimated according to ACI Sec. 11.2 and Sec. 8.5.1, respectively.

$$\sigma_{cr} = 6.7\sqrt{f'_{c}} \quad (7.3)$$

$$E = 57,000\sqrt{f'_{c}} \quad (7.4)$$

where $f'_{c}$ is the ultimate uniaxial cylinder compressive strength in psi. Cracking tensile strain can then be calculated from Eqs. (7.3) and (7.4) as follows

$$\varepsilon_{cr} = \frac{\sigma_{cr}}{E} \quad (7.5)$$
Tensile stress strain model can be obtained directly from direct tension test. However, the test procedure is rather time consuming and difficult to performed. In addition, using uniaxial tensile response normally underpredicts flexural strength for strain-softening FRC due to the size effect caused by the difference of uniform stress in direct tension test and gradient stress in bending test. To predict flexural behaviors, tensile properties obtained from back calculation of load-deflection curve of bending test are preferable. The material parameters obtained by this procedure already compensate the discrepancy between the two test methods such that they can better predict the strength of flexural members.

Statistical distribution of defects in granular materials of different volumes can also cause size effect. Using original RILEM design method to predict experimental results of structural elements of various sizes revealed that the model overpredicted the strength of large structures. For that reason the RILEM committee proposed the size-dependent safety factor \( \kappa_h \) to compensate this size effect (Vandewalle et al. 2003).

\[
k_h = \begin{cases} 
1.0 & h \leq 5 \text{ in} \\ 
1.0 - \frac{6(h - 5)}{190} & 5 \text{ in} < h \leq 24 \text{ in} \\ 
0.4 & h > 24 \text{ in} 
\end{cases} \tag{7.6}
\]

where \( h \) is the full height of the beam designed in inches. The guideline adopts this factor to account for the size effect in beam depth greater than 5 inches, in which the postpeak tensile strength \( \sigma_p \) is decreased by a factor of \( \kappa_h \). For uniformity of reliability index used in determining uniaxial cylinder compressive strength test stipulated by ACI Sec.5.3.2.1,
the required mean postpeak tensile strength of concrete $\sigma_{pm}$ obtained from the test must be at least

$$\sigma_{pm} = \frac{\sigma_p}{\kappa_h} + 1.34s$$  \hspace{1cm} (7.7)

where $s$ is the standard deviation from at least 30 samples and $\sigma_p$ is the specified postpeak tensile strength used in design calculations. The final postpeak requirement from material testing is shown as the dash line in Fig. 7.1(a).

7.2.1.2. Compression Model

Compressive response of strain-softening FRC is assumed to be elastic perfectly plastic as depicted in Fig. 7.1(b). Stress increases from zero to the compressive yield stress $\sigma_{cy}$ at compressive yield strain $\varepsilon_{cy}$, then remains constant until it is terminated at the ultimate compressive strain $\varepsilon_{cu}$. From material point of view, the compressive modulus $E_c$ can be different from the tensile modulus $E$ by a factor of $\gamma$;

$$\gamma = \frac{E_c}{E}$$  \hspace{1cm} (7.8)

However, FRC is commonly assumed that the tensile and compressive modulus are the same, which can be approximated by Eq. (7.4); thus, $E = E_c$ and $\gamma = 1$. This assumption was used in the derivation of strain-softening FRC, thus the restriction $\gamma$ must be 1. On the other hand, the assumption that $E$ can be different from $E_c$ was used in the derivation of hybrid reinforced concrete system, thus $\gamma$ can be any value. Another two normalized material parameters for the compression model are defined as follows

$$\omega = \frac{\varepsilon_{cy}}{\varepsilon_{cr}} = \frac{\sigma_{cy}}{E_c \varepsilon_{cr}} = \frac{\sigma_{cy}}{\gamma E \varepsilon_{cr}} = \frac{\sigma_{cy}}{\gamma \sigma_{cr}}$$  \hspace{1cm} (7.9)
\[ \lambda_{cu} = \frac{\varepsilon_{cu}}{\varepsilon_{cr}} \]  

(7.10)

where \( \omega \) is the normalized compressive yield strain and \( \lambda_{cu} \) is the normalized ultimate compressive strain. Note that for strain-softening FRC system that \( E = E_c \), \( \omega \) can be considered as compressive to tensile strength ratio. Material parameters from the RILEM model are adopted here (Vandewalle et al. 2003). The compressive yield stress \( \sigma_{cy} \) is 0.85\( f_c' \). The ultimate compressive strain \( \varepsilon_{cu} \) is taken as 0.0035, which is the lower bound value of typical SFRC (Swamy and Al-Ta’an 1981; Hassoun and Sahebjam 1985).

### 7.2.1.3. Steel Model

Steel reinforcement can be added to FRC system to increase load carrying capacity of structural members. Since the design equations are derived explicitly for a single reinforced FRC beam having a rectangular cross section, tension steel must be placed in tension side only. Compression steel can be added later in the final design just to provide a cage for hooking up stirrups. Fig. 7.1(c) shows the elastic perfectly plastic steel model that stress increases from zero to the steel yield stress \( f_{sy} \) at yield strain \( \varepsilon_{sy} \), then stress remains constant afterward. Two normalized material parameters for steel model are defined as follows:

\[ n = \frac{E_s}{E} \]  

(7.11)

\[ \kappa = \frac{\varepsilon_{sy}}{\varepsilon_{cr}} = \frac{f_{sy}}{E_s \sigma_{cr}} = \frac{f_{sy}}{n \sigma_{cr}} = \frac{f_{sy}}{n E \varepsilon_{cr}} \]  

(7.12)

where \( n \) is the modulus ratio between steel modulus \( E_s \) and FRC tensile modulus \( E \), \( \kappa \) is the normalized steel yield strain. Fig. 7.1(d) shows a reinforced FRC section and the
location of steel rebar \( d \). Two normalized geometric parameters for the reinforced section are defined as follows:

\[
\rho_g = \frac{A_s}{bh} \quad (7.13)
\]

\[
\alpha = \frac{d}{h} \quad (7.14)
\]

where \( \rho_g \) is the reinforcement ratio per gross sectional area \( bh \), not an effective area \( bd \) as commonly used in reinforced concrete nomenclature, \( A_s \) is the tension steel and \( \alpha \) is the normalized steel location relative to the beam depth \( h \).

### 7.2.2. Ultimate Moment Capacity

According to ultimate strength design philosophy, a structural member must be designed such that member capacity reduced by a reduction factor is still greater than internal forces due to factored loads that accounts for probability of overloading. Strength of the member can be determined by the proposed design equation and the internal forces can be obtained by linear elastic analysis using factor load coefficients according to ACI Sec 9.2.

#### 7.2.2.1. Fiber Reinforced Concrete System

Nominal moment capacity \( M_n \) can be estimated by the simplified equation derived in Chapter 5.

\[
\phi_{r1} M_n \geq M_u \quad (7.15)
\]

\[
M_n = M_n^{'} M_{cr} \quad (7.16)
\]

\[
M_n^{'} = \frac{3\omega \mu}{\omega + \mu} \quad (7.17)
\]
where $M_{cr}$ is the cracking moment of an FRC section. From Chapter 6, $\phi_1$ was identified to be 0.743 based on small sample size. A more conservative value $\phi_1 = 0.70$ is suggested here as the value obtained from small sample size is less reliable. This factor is subjected to change when more compressive test data become available. For convenience in the design process, Eqs. (7.16) and (7.17) were substituted into Eq. (7.15) and solved for the required postpeak tensile strength to meet the required ultimate moment as

$$\mu \geq \frac{M_u \omega}{3\omega \phi_1 M_{cr} - M_u}$$  \hspace{1cm} (7.19)

In conventional reinforced concrete structures, the minimum flexural reinforcement must be provided in order to avoid a sudden failure of a reinforced concrete beam after cracking. This requirement is implemented in the FRC system by providing sufficient amount of postpeak tensile strength $\mu$ such that there is no drop in flexural strength after crack initiation. In other word, FRC must be strain-softening-deflection-hardening. As derived in Chapter 5, the critical postpeak tensile strength $\mu_{crit}$ that changes flexural response from deflection-softening to deflection-hardening response is

$$\mu_{crit} = \frac{\omega}{3\omega - 1}$$  \hspace{1cm} (7.20)

For typical FRC materials, compressive to tensile strength ratio $\omega$ varies between 6 and 12. Thus, $\mu_{crit}$ varies in a narrow range between 0.353 and 0.343. Since the actual tensile strength and the strength used in the design calculations may be different by a
factor of \( \frac{7.5 \sqrt{f_c}}{6.7 \sqrt{f_c}} = 1.12 \), a slightly conservative value that ensures the deflection-hardening behavior of an actual flexural member is proposed as

\[
\mu_{\text{min, flex}} = 0.40 \tag{7.21}
\]

### 7.2.2.2. Hybrid Reinforced Concrete System

Flexural design equations for the hybrid reinforced concrete system are more comprehensive than those equations for FRC system presented in the previous section. The compressive modulus \( E_c \) can be set to a different value from the tensile modulus \( E \) and the section can be reinforced with flexural steel to increase the moment capacity. The reduced nominal moment capacity of the hybrid system is proposed as

\[
\phi_{r2} M_n \geq M_u \tag{7.22}
\]

\[
M_n = M'_n M_{cr} \tag{7.23}
\]

\[
M'_n = \frac{6 \rho_g n k (\mu \alpha - \mu + \alpha \gamma \omega) + 3 (\gamma \omega \mu - (\rho_g n k)^2)}{\gamma \omega + \mu} \tag{7.24}
\]

\[
M_{cr} = \frac{\sigma_{cr} bh^2}{6} \tag{7.25}
\]

where \( M_{cr} \) is the cracking moment of the plain FRC. The reduction factor \( \phi_{r2} \) for hybrid system identified in the previous chapter was 0.862. However, a slightly higher value \( \phi_{r2} = 0.90 \) as being used for reinforced concrete beams according to ACI Sec. 9.3.2.1 is adopted here. A small difference of these two numbers is attributed to the small sample size of 6 beam series used in statistically analysis to obtain the reduction factor. It should be pointed out that \( M'_n \) presented in Eq. (7.24) is reduced to \( M'_n \) presented in Eq. (7.17)
when the beam section does not contain flexural reinforcement \( (\rho_g = 0) \) and modulus of FRC are equal in compression and tension \( (\gamma = 1) \).

Due to the assumption used in the simplification, \( M_n^* \) defined by Eq. (7.24) is applicable for the members that fail in ductile manner only. Therefore, the reinforcement ratio \( \rho_g \) used in the equation must be less than the reinforcement ratio at balance failure \( \rho_{g,\text{bal}} \) given by

\[
\rho_{g,\text{bal}} = \frac{2\mu(2\lambda_{cu}(\alpha - 1) + \alpha - \kappa) + \alpha\gamma\omega(2\lambda_{cu} - \omega) - \alpha}{2n\kappa(\lambda_{cu} + \kappa)}
\]  

(7.26)

Similar to conventional reinforced concrete beams, the minimum flexural reinforcements is needed to prevent sudden failure after the reinforced beam cracks. The analytical minimum reinforcement derived in Chapter 6 was slightly increased by a safety factor of 3/2 and proposed for the design as

\[
\rho_{g,\text{min, flex}} = \frac{3G_1 - \sqrt{G_2 + 9G_1^2}}{2n\kappa}
\]  

(7.27)

where \( G_1 = \mu(\alpha - 1) + \alpha\gamma\omega \) and \( G_2 = 3\gamma\omega(3\mu - 1) - \mu \). As presented in Fig. 6.12(a) of Chapter 6 that the proposed minimum flexural reinforcement ratio approximately passed through the mean experimental results of beams at various sizes. With the increase in the reinforcement ratio by 3/2, it can assure that the flexural responses of all beam sizes will behave in ductile manner after cracking.

### 7.2.3. Minimum Postpeak Tensile Strength for Shrinkage and Temperature

Beside strength requirement for flexural loading, it is important for engineers to check if the performance of structural members satisfies serviceability criteria such as crack width
limit. Due to phase change of FRC from liquid to solid, large surface to volume ratio of reinforced concrete slab can cause severe cracking due to shrinkage. In order to control crack width within acceptable level, the minimum shrinkage and temperature reinforcement must be placed perpendicular to the main flexural reinforcement. According to the ACI Sec. 7.12.2.1, the minimum reinforcement ratio per gross section area is

\[
\rho_{g, min, shrk} = \begin{cases} 
0.0020 & \text{for Grade 40 and 50 deformed bar} \\
0.0018 & \text{for Grade 60 or welded-wire fabric (smooth or deformed)} \\
\frac{0.0018 \times 60,000}{f_{sy}} & \text{but not less than 0.0014} \\
\end{cases}
\]

(7.28)

for the steel having yield strength greater than 60 ksi at yield strain of 0.35%

When steel reinforcement is replaced with FRC, the minimum normalized postpeak tensile strength \(\mu_{min, shrk}\) to control crack and shrinkage is determined based on the equivalent tensile capacity and probability of failure.

\[
\phi_b \rho_{min, shrk} f_{sy} bh = \phi_{r1} \mu_{min, shrk} \sigma_{cr} bh
\]

(7.29)

It is assumed that the reduction factor for the members that the failure is controlled by the reinforcement is the same as the factor used for bending of reinforced concrete beam, \(\phi_b = 0.90\). The reduction factor for the members that the failure is governed by the tensile capacity of FRC is the same as the factor used for predicting flexural strength of FRC, \(\phi_{r1} = 0.70\). Conservative values \(\rho_{g, min, shrk} = 0.0018\) and \(f_{sy} = 60,000\) psi are substituted in Eq. (7.28) to produce highest tension force and solve for \(\mu_{min, shrk}\)
\[
\mu_{\text{min,shrk}} = \frac{140}{\sigma_{cr}} \quad (7.30)
\]

where \(\sigma_{cr}\) is the cracking tensile strength in psi.

### 7.2.4. Shear Strength Capacity

The original shear strength equation for reinforced concrete beams according to ACI Sec. 11.3.1.1 is empirically presented as a function of the cylinder concrete compressive strength \(f'_{c}\) and beam web \(b_w\) of a T- or rectangular beam as

\[
V_c = 2\sqrt{f'_{c}b_wd} \quad (7.31)
\]

Since the design equations presented in this chapter is applicable to rectangular cross section only and the full depth of FRC contributes to the shear strength of members. Thus the original ACI equation is modified for FRC as follows:

\[
\phi_vV_{FRC} = \phi_v2\sqrt{f'_{c}bh} \quad (7.32)
\]

where \(\phi_v = 0.75\) is the reduction factor for shear according to ACI Sec. 9.3.2.3; \(V_{FRC}\) is the shear strength of FRC section in lb; \(f'_{c}\) is in psi; \(b\) and \(h\) are in inches; It can be seen that a small gain in shear strength by a factor of \(h/d\) is obtained by Eq. (7.32). This is somewhat conservative since the shear strength of deflection-hardening FRC (\(\mu_{\text{min,flex}} > \mu_{\text{crit}}\)) is expected to be much higher than the shear strength of plain concrete.

When steel reinforcement is used in the hybrid system, shear capacity is enhanced by mean of dowel action and crack width control. Parra-Montesinos (2006) conducted extensive shear test of hybrid beams using SFRC and steel rebar, and proposed the empirical shear strength equation as follows.
\[ \phi_v V_{SFRC} = \phi_v 3.50 \sqrt{f_{c'} b d} \]  

(7.33)

where \( V_{SFRC} \) is the shear strength of the hybrid section in lb; \( f_{c'} \) is in psi; \( b \) and \( h \) are in inches; The applicability of Eq. (7.33) is restricted to the range of variables used in the test and the observation made from the experimental results

Section must contain steel reinforcement

Fiber types must be hooked or crimped steel fibers

Fiber volume fraction \( V_f \) must greater than 0.75%

Range of cylinder concrete compressive strength, 2.6 ksi < \( f_{c'} \) < 15.1 ksi

Range of effective beam depth, 7 in. < \( d \) < 22.5 in.

Range of shear span-to-effective depth ratio, 1 < \( X/d \) < 6

It is important for engineers to understand the nature of empirical shear Eqs. (7.32) and (7.33) obtained from beam test results. The shear capacity is primarily contributed from the uncracked portion of the compressive zone and much less from the cracked tensile zone. Even thought discrete fibers can significantly increase postcrack tensile capacity of the concrete matrix, the minimum shear reinforcement stipulated by ACI building codes shall not be totally replaced with FRC in the situations that load reversal can happen, such as structures in seismic zones or irregular structures that bending moment can flip direction due to movement of live loads. In these cases, cracks appear at the top face as well as the bottom faces and the shear capacity of the fully cracked section is almost diminished. Without the presence of shear reinforcement, the structures can collapse in a few load cycles.
Generally speaking, only structures in non seismic zone that are dominated by dead load and their relative beam spans are not varied greatly such that no pattern loading is required in the analysis are good candidates for using FRC without shear reinforcement. Otherwise, the minimum shear reinforcement is still enforced to control crack width and provide adequate shear resistance.

7.2.5. Material Testing

Several test methods have been proposed in characterization of postpeak tensile capacity of FRC. The merit and drawback were discussed in details by Gopalaratnam (1995). One of the most practical method is the Averaged Residual Strength (ARS) proposed by Banthia and Dubey (1999, 2000), in which the postpeak tensile strength can be obtained by a simple open loop testing machine (ASTM C-1399 1999). First, a steel plate is placed underneath a concrete beam specimen and loaded under four-point bending set up until concrete cracks. Then, the steel plate is removed and the cracked specimen is reloaded to obtain postpeak flexural strengths at deflection levels of 0.02, 0.03, 0.04 and 0.05 inches. Finally the strengths are averaged to represent an averaged residual strength (ARS value). It is very important to mention that the ARS value is an equivalently elastic stress, which is not the same as the postpeak tensile strength $\sigma_p$ in Eq. (7.1) and Fig. 7.1(a).

Soranakom and Mobasher proposed Excel worksheet to ACI committee 544 (2005) to be used in back calculation of tensile properties from the load-deflection response of three- or four- point bending test. The worksheet is also applicable for the ARS test method by considering only the reloading response of a cracked beam. For each load-deflection, the normalized postpeak tensile strength $\mu$ is determined by trial and error.
process until the predicted postpeak response best fit to the experimental results at deflection levels between 0.02 in. and 0.05 in.

Material properties obtained from testing may be different from those used in the design calculations that were estimated by the relationship to the compressive strength $f_{c'}$. As strength of the designed members is concerned, only the postpeak tensile strength $\sigma_p$ obtained from the test is important. It must meet the specified strength $\sigma_p$ used in design calculations. Small discrepancy between the estimated values used in the design calculations and the actual values obtained from testing are indirectly taken care by the redundant safety factors compounded in the design process.

### 7.2.6. Design Examples

The design guidelines for strain-softening FRC are best suit for slab and footing systems. These thin structures are less sensitive to the size effect that decreases the strength of materials. Three examples of slab on grade, four-span floor slab and wall footing are given here to demonstrate the design calculations. Various aspects of structural concrete design may require readers to consult reinforced concrete text books using ACI approach.

#### 7.2.6.1. Example 1: Slab on Grade

A five inch thick reinforced concrete slab has steel rebar #4@18 reinforced at mid depth. The materials used are: concrete compressive strength $f_{c'}$ of 3,000 psi and steel yield strength $f_{sy}$ of 60 ksi. Replace this existing design with SFRC that has compressive strength $f_{c'}$ of 4000 psi

Typically, slabs on grade are designed based on minimum shrinkage and temperature steel. Loads on slab are not critical and normally transferred directly to the stiff
compacted soils. These slabs are allowed to crack but not to disintegrate. Other types of slab on grade and pavement that are designed based on the loading conditions and subgrade modulus on are not considered here.

**Moment capacity of five inch thick reinforced concrete slab**

The slab is designed based on 1 foot strip and the amount of reinforcement $A_s$ is calculated by

$$A_s = \pi d^2 \frac{12}{4 \text{ Spacing}} = \pi 0.5^2 \frac{12}{4 \times 18} = 0.131 \text{ in}^2/\text{ft}$$

Calculate the length of plastic compressive zone according to ACI stress block concept

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.131 \times 60}{0.85 \times 3 \times 12} = 0.257 \text{ in.}$$

According to ultimate strength design philosophy, the ultimate moment is equal to the reduced nominal moment capacity

$$M_u = \phi_b M_n = \phi_b A_s f_{sy} \left(d - \frac{a}{2}\right)$$

$$= 0.9 \times 0.131 \times 60 \left(2.5 - \frac{0.257}{2}\right) \frac{1}{12} = 1.40 \text{ kips-ft/ft}$$

**Replace the moment capacity with SFRC, $f'_c = 4000$ psi**

Calculate the cracking tensile strength of SFRC according to Eq. (7.3)

$$\sigma_{cr} = 6.7 \sqrt{f'_c} = 6.7 \sqrt{4000} = 424 \text{ psi}$$

Calculate the cracking moment according to Eq. (7.18)

$$M_{cr} = \frac{\sigma_{cr} bh^2}{6} = \frac{424 \times 12 \times 5^2}{6 \times 12000} = 1.77 \text{ kips-ft/ft}$$
For strain-softening FRC without flexural reinforcement, compressive and tensile modulus are equal; i.e., \( \gamma = E_c/E = 1 \). The normalized compressive yield strain can be obtained by Eq. (7.9)

\[
\omega = \frac{\sigma_{cy}}{\gamma \sigma_{cr}} = \frac{0.85 \times 4000}{1 \times 6.7 \sqrt{4000}} = 8.02
\]

Determine the normalized postpeak tensile strength for FRC system by Eq. (7.19)

\[
\mu = \frac{M_u \omega}{3 \omega \phi_{r1} M_{cr} - M_u} = \frac{1.40 \times 8.02}{3 \times 8.02 \times 0.7 \times 1.77 - 1.40} = 0.395
\]

The calculated \( \mu \) is less than the minimum postpeak tensile strength for flexure defined by Eq. 7(21), \( \mu_{min,\text{flex}} = 0.40 \) (governed). Thus, \( \mu \) must be at least 0.40.

**Replace tensile capacity**

Plain concrete in reinforced concrete slab has no residual tensile strength; therefore, only the amount of reinforcement will be replaced with SFRC to provide the same tensile capacity. It is assumed that the reduction factor \( \phi_b \) is used for the member that fails by tensile steel and the reduction factor \( \phi_{r1} \) is used for member that fails by postpeak tensile strength of SFRC. The equivalent of tensile strength can be expressed as

\[
\phi_b A_s f_{sy} = \phi_{r1} \mu \sigma_{cr} bh
\]

The postpeak tensile strength is then calculated

\[
\mu = \frac{\phi_b A_s f_{sy}}{\phi_{r1} \sigma_{cr} bh} = \frac{0.9 \times 0.131 \times 60}{0.7 \times 0.424 \times 12 \times 5} = 0.397 \quad \Rightarrow \text{(not critical)}
\]
**Minimum normalized postpeak tensile strength for shrinkage and temperature**

For serviceability criterion, the minimum normalized postpeak tensile strength can be calculated by Eq. (7.30)

\[
\mu_{\text{min,shrk}} = \frac{140}{\sigma_{cr}} = \frac{140}{424} = 0.330 \quad \Rightarrow \text{(not critical)}
\]

**Discussions**

If the goal is to replace the existing reinforced concrete slab with SFRC and still have the same performance and reliability index, the postpeak tensile strength must be 0.40 based on the minimum postpeak tensile strength for flexure. However, if only shrinkage and temperature is of concern, the required strength for slab on grade can be reduced to 0.330. For this problem, specify \( \mu = 0.40 \) based on equivalent performance. It should be pointed out that the specified strength must be denormalized by the same cracking tensile strength \( \sigma_{cr} \) used in the design calculations, which may be different from the actual value obtained from material testing. According to the definition in Eq. (7.1)

\[
\sigma_p = \mu \sigma_{cr} = 0.40 \times 424 = 170 \text{ psi}
\]

**7.2.6.2. Example 2: Four-Span Floor Slab**

Design a four-span floor slab rested on steel stringers, subjected to a live load of 40 psf and super imposed dead load of 10 psf as shown in Fig. 7.2. Use SFRC that has compressive strength \( f'_c \) of 4000 psi and ignore the width of steel stringers in calculation of clear span between each support. The SFRC has unit weight of 150 pcf.

The floor slabs will be designed as a one way slab based on 1 foot strip. Assume that thickness of SFRC slab can be estimated in the same way as reinforced concrete beam
according to ACI Sec. 9.5.2.1, Table 9.5(a). The critical section is at the end span (8 ft) and the thickness is determined by

\[ h = \frac{L}{24} = \frac{8 \times 12}{24} = 4 \text{ in.} \]

Estimate self weight of SFRC floor slab

\[ w_{sw} = \frac{4}{12} \times 150 = 50 \text{ lb/ft}^2 \]

Calculate factored load on the slab according to ACI Sec. 9.2.1

\[ w_u = 1.2(DL) + 1.6(LL) \]

\[ w_u = 1.2(50 + 10) + 1.6(40) = 136 \text{ lb/ft}^2 \]

**Moment Capacity**

Since the live load is less than three times of the dead load, adjacent spans are not different by more than 20% and other requirements of ACI Sec. 8.3.3 are met, the ACI moment coefficients can be used to calculate the ultimate moment at the critical section, which is the first interior support.

\[ M_u = C_m w_u L^2 = \frac{1}{10} \times 0.136 \times 8^2 = 0.87 \text{ kips-ft/ft} \]

The same material properties of SFRC used in Example 1 are: the cracking tensile strength \( \sigma_{cr} \) of 424 psi and the normalized compressive yield strain \( \omega \) of 8.02. For 4 inch thick section, the cracking moment is calculated by Eq. (7.18)

\[ M_{cr} = \frac{\sigma_{cr} bh^2}{6} = \frac{424 \times 12 \times 4^2}{6} \times \frac{1}{12000} = 1.13 \text{ ft-kip} \]

The postpeak tensile strength for the ultimate moment \( M_u \) is determined by Eq. (7.19)
The minimum postpeak tensile strength for flexure defined by Eq. (7.21), $\mu_{\text{min,flex}}$ is 0.40 and the minimum for shrinkage and temperature defined by Eq. (7.27), $\mu_{\text{min,shrk}}$ is 140/424 = 0.330. Thus, $\mu = 0.40$ is govern the design. The specified postpeak tensile strength for this problem is calculated by Eq. (7.1)

$$\sigma_p = \mu \sigma_{cr} = 0.40 \times 424 = 170 \text{ psi}$$

**Shear Capacity**

In general shear is not a concern in the design of floor slabs. However, the calculations are shown here to illustrate the design procedure. The critical shear location is at the first interior support, which can be determined by coefficient method ACI Sec. 8.3.3.

$$V_u = C_v \left( \frac{w_u L}{2} \right) = 1.15 \times \frac{0.136 \times 8}{2} = 0.623 \text{ kip/ft}$$

Check the shear capacity of SFRC without flexural reinforcement by Eq. (7.32)

$$\phi \cdot V_{FRC} = \phi_v \cdot 2 \sqrt{f'_c} \cdot bh = 0.75 \left( \frac{2 \sqrt{4,000 \times 12 \times 4}}{1000} \right) = 4.55 \text{ kips} >> V_u \text{ of } 0.623 \text{ kips}$$

Therefore, the slab has adequate shear capacity and the specified postpeak tensile strength $\sigma_p = 170 \text{ psi}$ is adequate.

**7.2.6.3. Example 3: Wall Footing**

Design the wall footing shown in Fig. 7.3 subjected to a dead load of 10 kips/ft and a live load of 12 kips/ft along the length of the wall. The thickness of concrete wall is 1 foot. The allowable soil pressure $q_a$ at 5 feet below the top soil is 5,000 psi. Use SFRC with a
compressive strength $f_c'$ of 4,000 psi and steel with a yield strength $f_{sy}$ of 60,000 psi and Young’s modulus $E_s$ of $29 \times 10^6$ psi. The density of the SFRC and soil are 150 pcf and 120 pcf, respectively.

**Size of footing**

Design a footing in short span direction based on 1 foot strip. According to ACI Sec. 15.7, the effective depth of footing on soil shall not be less than 6 inches. Thus, the preliminary thickness of the footing is assumed to be 9 inches, allow 3 inches for concrete cover. This assumed thickness will be checked later. The net allowable soil pressure at 5 feet after subtracting the weight of footing and soil is

$$q_n = 5 - \left(\frac{9}{12} \times 0.150 + (5 - \frac{9}{12}) \times 0.120\right) = 4.38 \text{ ksf}$$

Determine the width of the footing $B$ based on net allowable soil pressure that can carry unfactored dead load and live load

$$B = \frac{DL + LL}{q_n} = \frac{10 + 12}{4.38} = 5.02 \text{ ft}$$

Let use $B = 5$ feet. In ultimate state, the factored net soil pressure is

$$q_{nu} = \frac{1.2(DL) + 1.6(LL)}{B \times L} = \frac{1.2 \times 10 + 1.6 \times 12}{5 \times 1} = 6.24 \text{ ksf}$$

Thickness of footing is normally governed by the shear capacity of reinforced concrete section. When SFRC is used with conventional steel, shear strength Eq. (7.33) is applied. SFRC can reduce crack width significantly such that the stringent requirement for concrete cover of 3 inches [ACI Sec. 7.7.1] for concrete contact to soil seems to be excessive. Smaller clear cover can be used with engineering judgment. In this case clear
cover of 2 inches and bar diameter of 6/8” are selected for the footing. The effect depth is then calculated as

\[ d = 9 - 2 - 0.5 \times \frac{6}{8} = 6.625 \text{ in.} \] greater than the minimum of 6 in. => OK

Shear capacity

As shown in the Fig. 7.3, the critical shear section is at a distance \( d \) from the edge of the wall; thus the ultimate shear load is calculated as:

\[ V_u = 6.24 \left( \frac{5 - 1}{2} - \frac{6.625}{12} \right) = 9.04 \text{ kips/ft} \]

Check if the reinforced SFRC section has adequate shear capacity by Eq. (7.33)

\[ \phi V_{SFRC} = \phi V 3.50 \sqrt{f'_c b d} = 0.75 \left( \frac{3.5 \times 1000 \times 12 \times 6.625}{1000} \right) = 13.2 \text{ kips/ft} \]

This section has an adequate shear capacity, greater than the shear load \( V_u \) of 9.04 kips/ft; therefore, the assumed depth of 9 inches is adequate.

Moment capacity

As shown in the Fig. 7.3, the ultimate moment is critical at the edge of wall and calculated by

\[ M_u = 6.24 \times \frac{2.0^2}{2} = 12.5 \text{ kips-ft/ft} \]

In order to determine the moment capacity of the hybrid reinforced concrete footing, several parameters are needed to determine as follows:

Cracking tensile strength by Eq. (7.3), \( \sigma_{cr} = 6.7 \sqrt{f'_c} = 6.7 \sqrt{4000} = 424 \text{ psi} \)

Young’s modulus of concrete by Eq. (7.4),
\[ E = 57,000 \sqrt{f_c} = 57,000 \sqrt{4000} = 3.605 \times 10^6 \text{ psi} \]

Cracking tensile strain by Eq. (7.5),
\[ \varepsilon_{cr} = \frac{\sigma_{cr}}{E_c} = \frac{424}{3.605 \times 10^6} = 1.176 \times 10^{-4} \]

Assume compressive and tensile modulus of concrete are the same by Eq. (7.8), \( \gamma = 1 \)

Normalized compressive yield strain by Eq. (7.9),
\[ \omega = \frac{\sigma_{cy}}{\gamma \sigma_{cr}} = \frac{0.85 \times 4000}{1 \times 424} = 8.02 \]

Normalized ultimate compressive strain from Eq. (7.10),
\[ \lambda_{cu} = \frac{\lambda_{cu}}{\varepsilon_{cr}} = \frac{0.0035}{1.176 \times 10^{-4}} = 29.8 \]

Modulus ratio by Eq. (7.11),
\[ n = \frac{E_s}{E} = \frac{29 \times 10^6}{3.605 \times 10^6} = 8.04 \]

Normalized steel yield strain by Eq. (7.12),
\[ \kappa = \frac{\varepsilon_{sy}}{\varepsilon_{cr}} = \frac{f_{sy}}{n \sigma_{cr}} = \frac{60,000}{8.04 \times 424} = 17.6 \]

Normalized reinforcement depth by Eq. (7.14),
\[ \alpha = \frac{d}{h} = \frac{6.625}{9} = 0.74 \]

If SFRC is intended to be used to control crack width due to shrinkage and temperature in the direction perpendicular to the main flexural reinforcement, the normalized postpeak tensile strength can be calculated by Eq. (7.30)
\[ \mu_{min,shrk} = \frac{140 \text{ psi}}{\sigma_{cr}} = \frac{140}{424} = 0.330 \]

To design a ductile flexural members, the reinforcement ratio must be less than the ratio at balance failure calculated by Eq. (7.26).
\[ \rho_{g,bal} = \frac{2 \mu(\lambda_{cu} - \alpha - \kappa) + \alpha \gamma \omega(2 \lambda_{cu} - \omega)}{2 n \kappa (\lambda_{cu} + \kappa)} \]
\[ M_{cr} = \frac{\sigma_{cr} bh^2}{6} = \frac{424 \times 12 \times 9^2}{6 \times 12000} = \frac{1}{5.72} \text{ kips-ft/ft} \]

For the required ultimate moment, the reinforcement ratio can be determined by Eqs. (7.22) - (7.24)

\[ M_u = \frac{6 \rho_g n \kappa (\mu \alpha - \mu + \alpha \gamma \omega) + 3 \left( \gamma \omega \mu - \left( \rho_g n \kappa \right)^2 \right)}{\gamma \omega + \mu} M_{cr} \]

\[ 6 \times \rho_g \times 8.04 \times 17.6 (0.33 \times 0.74 - 0.33 + 0.74 \times 1 \times 8.02) \]

\[ + 3 \left( 1 \times 8.02 \times 0.33 - \left( \rho_g \times 8.04 \times 17.6 \right)^2 \right) \]

\[ 12.5 = 0.9 \frac{1 \times 8.02 + 0.33}{5.72} \]

\[ -37244 \rho_g^2 + 2736 \rho_g - 8.278 = 0 \]

By solving the quadratic equation, two solutions for \( \rho_g = 0.0032 \) and 0.0703 are obtained. Since the design equation is applicable for ductile failure only, the first solution \( \rho_g = 0.0032 < \rho_{g, bal} = 0.0216 \) is the correct solution.

**Check the minimum flexural reinforcement**

Calculate coefficients \( G_1 \) and \( G_2 \) for the minimum flexural reinforcement defined in Eq. (7.27)

\[ G_1 = \mu (\alpha - 1) + \alpha \gamma \omega = 0.33 (0.74 - 1) + 0.74 \times 1 \times 8.02 = 5.849 \]
\( G_2 = 3(\gamma \omega (3\mu -1) - \mu) = 3(1 \times 8.02(3 \times 0.33 -1) - 0.33) = -1.231 \)

\[
\rho_{g,\text{min,flex}} = \frac{3G_1 - \sqrt{G_2 + 9G_1^2}}{2n\kappa} = \frac{3 \times 5.849 - \sqrt{-1.231 + 9 \times 5.849^2}}{2 \times 8.04 \times 17.6} = 0.000124
\]

The calculated reinforcement ratio \( \rho_g = 0.0032 \) is greater than the minimum for shrinkage and temperature, \( \rho_{g,\text{min,flex}} = 0.000124 \). Thus, use \( \rho_g = 0.0032 \) to calculate the steel area \( A_s = 0.0032 \times 12 \times 9 = 0.346 \) in\(^2\). Use bar #6@15” (\( A_s = 0.353 \) in\(^2\)/ft). The postpeak tensile strength is calculated by Eq. (7.1)

\[
\sigma_p = \mu \sigma_{cr} = 0.33 \times 424 = 140 \text{ psi}
\]

**Summary**

The designed footing is 5 feet wide and 9 inch thick. Steel rebar #6@15” is placed in the short span direction at a depth of 6.625 inches. The specified postpeak tensile strength \( \sigma_p \) is 140 psi.

**7.2.7. Inverse Analysis of Load-Deflection Response**

This section explains how to use the Excel worksheet shown in Fig. 7.4 to perform back calculation of postpeak tensile strength from load-deflection response obtained from the ARS test method. Only some parts of the worksheet are presented here for brevity. The theoretical background and all computed parameters in the whole worksheet require readers to consult Soranakom and Mobasher (2007b). Users need to input the data in the highlighted cells as shown in Fig. 7.4(a). The worksheet will automatically calculate the parameters and show the predicted load-deflection response. The unit used is a consistent unit (N, mm, MPa for SI unit and lb, in, psi for English unit).
Example

Determine a postpeak tensile strength of a given SFRC mix using a load-deflection response of a beam sample obtained from ARS test method and verify that if this mix is qualified for the three design examples presented in the previous sections. From material testing, it is found that the mean compressive strength $f_{cm}$ of three samples is 5,000 psi, which is higher than the specified strength $f'_c$ of 4,000 psi used in the design calculations. The dimensions of the beam sample 1 are 4.03 in. x 4.06 in. x 14 in. with a clear span of 12 in. A standard deviation of the postpeak tensile strength from the manufacture database is known to be 50 psi.

With this information, enter: test method = 4 (for four point bending test), $b = 4.03$, $h = 4.06$, $L = 12$, ignore $L_p$ (plastic length is required for three-point bending test only). Other input parameters can be calculated from the relationship to the mean compressive strength $f_{cm}$ (not $f'_c$) or obtain directly from the tests or combinations as follows. Estimate Young’s modulus from the mean compressive strength by Eq. (7.4),

$$E = 57,000\sqrt{f_{cm}} = 57,000\sqrt{5,000} = 4.030 \times 10^6 \text{ psi}$$

Assume compressive modulus is the same as tensile modulus; thus, $\gamma = E_c/E = 1$ by Eq. (7.8). In contrast to the strength design, material properties used in back calculation process are aimed to predict the load-deflection response, not a conservative strength of design members. Therefore, the cracking tensile strength defined by Eq. (7.5) is replaced by a modulus of rupture used in the calculation of deflection defined by ACI Sec.9.5.2.3.

$$\sigma_{cr} = 7.5\sqrt{f_{cm}} = 7.5\sqrt{5,000} = 530 \text{ psi}$$
Calculate other input material parameters as follow:

Cracking tensile strain by Eq. (7.5), \( \varepsilon_{cr} = \frac{530}{4.03 \times 10^6} = 1.316 \times 10^{-4} \)

Normalized ultimate tensile strain by Eq. (7.2), \( \beta_{tu} = \frac{\varepsilon_{tu}}{\varepsilon_{cr}} = \frac{0.025}{1.316 \times 10^{-4}} = 190 \)

Compressive yield stress is estimated as \( \sigma_{cy} = 0.85 f_{cm} = 0.85 \times 5,000 = 4,250 \) psi

Normalized compressive yield strain by Eq. (7.9), \( \omega = \frac{\sigma_{cy}}{\gamma \sigma_{cr}} = \frac{4250}{1 \times 530} = 8.02 \)

Normalized ultimate compressive strain by Eq. (7.10), \( \lambda_{cu} = \frac{\varepsilon_{cu}}{\varepsilon_{cr}} = \frac{0.0035}{1.316 \times 10^{-4}} = 26.6 \)

All parameters for predicting load-deflection are defined, except the normalized postpeak tensile strength \( \mu \). By trial and error the value of \( \mu \) between 0 and 1 until the predicted response matches the experimental in the postpeak region at deflection level of 0.035 inches, the center of 4 deflection points used in the calculation of ARS value, as shown in Fig. 7.4(b). The final trial value, \( \mu = 0.435 \) is presented in Fig. 7.4(a). The calibrated tension and compression models are presented in Fig. 7.4(c and d), in which the postpeak tensile strength \( \sigma_p \) is 231 psi. When all three samples are analyzed, the average value and the standard deviation presented in Table 7.1 are 229 psi and 37.2 psi, respectively.

**Check if this mix is qualified for the three design examples**

Example 1: Slab on grade is 5 inch thick and the specified postpeak tensile strength \( \sigma_p \) is 170 psi. From Eq. (7.6), the size-dependent safety factor \( \kappa_h = 1.0 \). With the standard deviation of 50 psi and Eq. (7.7), and the mean postpeak tensile strength \( \sigma_{pm} \) obtained from the test must be at least
\[
\sigma_{pm} = \frac{\sigma_p}{\kappa_h} + 1.34s = \frac{170}{1.00} + 1.34(50) = 237 \text{ psi}
\]

Example 2: Four-span floor slab is 4 inch thick and the specified postpeak strength \( \sigma_p \) is 170 psi. The size-dependent safety factor \( \kappa_h \) is 1.0. Thus the mean postpeak tensile strength \( \sigma_{pm} \) is

\[
\sigma_{pm} = \frac{170}{1.00} + 1.34(50) = 237 \text{ psi}
\]

Example 3: Wall footing is 9 inch thick and the specified postpeak strength \( \sigma_p \) is 140 psi. The size-dependent safety factor, \( \kappa_h = 1.0 - 0.0316(h-5) = 1.0 - 0.0316(9-5) = 0.87 \). Thus, the mean postpeak tensile strength \( \sigma_{pm} \) is

\[
\sigma_{pm} = \frac{140}{0.87} + 1.34(50) = 228 \text{ psi}
\]

Summary

From the ARS test results analyzed by Excel worksheet, the mean postpeak tensile strength of 3 samples is 229 psi, which is greater than the minimum requirements for Example 3 only \( \sigma_{pm} \) = 228 psi. Therefore, the material is approved to be used in wall footing project of Example 3.

7.2.8. Discussions

The model predictions used in the guideline are based primarily on the simplified equations derived in Chapter 5 and 6 while the material properties are primarily adopted from ACI 318 and RILEM TC 162-TDF. The material mentioned in ACI is an aggregate based concrete while the one mentioned in RILEM is steel fiber reinforced concrete. For more reliable design method, specific types of FRC such as SFRC or GFRC or other
synthetic FRC shall be evaluated separately to identify the deficiency of the model predictions and additional provisions for particular systems may be amended to the general guideline presented in this chapter.

It can be noticed that the design procedure considers the size dependant safety factor $\kappa_h$ at the last step of calculations. This is done for the two reasons. First, it is more convenient to determine the size effect at the end rather than integrate the safety factor in the equations for calculating moment capacity, minimum postpeak tensile strength for flexure, minimum postpeak tensile strength for shrinkage and temperature and minimum flexural reinforcement. Second, the origin of the size dependent safety factor is not fully understood (Vandewalle et al. 2003) but adopted here to safeguard the model predictions for deep beams. Directly implement the correction factor from one model to the other model is not appropriate; however, it does provide reasonable safety factor due to beam size effect. For these two reasons, the size dependent safety factor is lump together with the standard deviation to obtain a final postpeak tensile strength to be used in the construction sites. A comprehensive experimental program of several beam sizes shall be conducted and calibrated with the proposed models to obtain its own size dependant safety factor for more reliable in strength prediction.

7.3. Strain-Hardening Fiber Reinforced Concrete

Unlike the strain-softening FRC that normally made of normal concrete and discrete fibers, material compositions and manufacturing techniques for strain-hardening FRC can varies greatly from one to another. For examples, SIMCON is made by infiltrating fine aggregate concrete into dense fiber mats to produce a superior tensile performance
material; ECC is made of mixing mortar with high volume polyvinyl alcohol (PVA) or polyethylene (PE) fibers to produce tough cement based matrix. The same acronym GFRC can be either strain-softening or strain-hardening depending on the matrix used and fiber contents. When low to moderate amount of glass fibers is added to normal concrete during mixing, it creates strain-softening GFRC that has decent postcrack tensile capacity. For some applications that require high tensile performance, GFRC is made of fine grain aggregate or mortar and high glass fiber dosages.

In this section the GFRC refers to the composites made of mortar or fine grain aggregate used in premix and spray method that normally contain high fiber dosages between 4% and 7% by weight. Depending on raw materials used and manufacturing techniques, producers can establish material properties to their design variables in a form of database. Table 7.2 shows a typical range of GFRC properties, which is used to establish the simplified design equations for strain-hardening GFRC in this chapter.

7.3.1. Material Models and Parameters

Full constitutive laws of strain-softening and hardening FRC presented in Fig. 5.11 in Chapter 5 are simplified to the models presented in Fig. 7.5. For typical tensile response of GFRC as presented in Fig. 7.5(a), the postcrack modulus $E_{cr}$ is relatively flat and the transition strain $\varepsilon_{trn}$ is relatively large compare to the cracking tensile strain $\varepsilon_{cr}$. With this unique characteristic, the flexural strength after cracking continues to increase and then drops after passing the strain at peak stress $\varepsilon_{trn}$. Thus, the tensile strain up to the strain at peak stress is sufficient to estimate the maximum moment capacity for the design. Therefore, the postpeak tensile response of the original tension model in Fig. 5.11(a) is
removed to obtain the simplified tension model for strain-hardening GFRC as shown in Fig. 7.5(a). In general, the compressive strength of GFRC is several higher than the tensile strength; thus, the flexural capacity is controlled by the weaker tension and the compressive stress developed in the beam section is still in elastic range. For this reason, the original elastic-perfectly plastic compression model presented in Fig. 5.11(b) is simplified to an elastic compression model presented in Fig. 7.5(b). The parametric studies of the original model shows that the difference in compressive and tensile modulus has little effect to the flexural strength prediction; thus, it assumes both modulus are the same \((E_c/E = \gamma = 1)\) in the simplified model. Since the compressive strength of GFRC can not be infinite, the stress is limited to the compressive yield stress \(\sigma_{cy} = 0.85f'_c\) at compressive yield strain \(\varepsilon_{cy}\). The equations for calculating the material parameters required for the simplified models are summarized as follow

\[
\varepsilon_{cr} = \frac{\sigma_{cr}}{E} \quad (7.34)
\]

Normalized tensile strain at peak stress, \(\alpha = \frac{\varepsilon_{tm}}{\varepsilon_{cr}} \quad (7.35)\)

Normalized postcrack tensile modulus, \(\eta = \frac{E_{cr}}{E} \quad (7.36)\)

Normalized compressive yield strain, \(\omega = \frac{\sigma_{cy}}{E\varepsilon_{cr}} = \frac{\sigma_{cy}}{\sigma_{cr}} \quad (7.37)\)

### 7.3.2. Ultimate Moment Capacity

Similar design philosophy used in the strain-softening material is adopted here. The nominal moment capacity of a member \(M_n\) is reduced by a reduction factor to account for
material variability and workmanship. The reduced capacity must exceed the ultimate moment $M_u$ due to factored loading.

$$\phi_{r3} M_n \geq M_u \quad (7.38)$$

The closed form solutions for moment and neutral axis in stage 2.1 of strain-hardening/softening presented in Table 5.10 and Fig. 5.(12.b.1) of Chapter 5 was simplified by assuming compressive modulus equal to tensile modulus ($\gamma = E_c/E = 1$) to obtain a simplified design equations for strain-hardening GFRC. The nominal moment capacity $M_n$ is expressed as a product of the normalized value $M_n'$ and cracking moment $M_{cr}$. This approach requires identifying the neutral axis parameter $k$. All equations for predicting flexural strength are presented as

$$M_n = M_n' M_{cr} \quad (7.39)$$

$$M_n' = C_2 \frac{k^2 - 2k + 1}{\beta^2} + \frac{2\beta k^3}{1-k} \quad (7.40)$$

$$M_{cr} = \frac{\sigma_{cr} b h^2}{6} \quad (7.41)$$

$$k = \frac{C_1 - \sqrt{\beta^2 C_1}}{C_1 - \beta^2} \quad (7.42)$$

where parameters $C_1 = \eta (\beta^2 - 2\beta + 1) + 2\beta - 1$ and $C_2 = C_1 + 2C_1\beta - \beta^2$; the normalized tensile strain at bottom fiber is defined as $\beta = \varepsilon_{tbot}/\varepsilon_{cr}$; $\phi_{r3}$ is a reduction factor for strain-hardening GFRC, which assumes to be 0.65, similar to the factor for compressive failure of plain concrete stipulated by ACI Sec. C.3.5. Despite the flexural response of GFRC is ductile that it can sustain large deflection after cracking, the postcrack response continues
to increase up to the ultimate moment and fails abruptly with little warning. For this reason, the same safety factor for compressive failure of plain concrete is applied here.

According to bilinear tension and elastic compression models depicted in Fig. 7.5(a and b), the moment capacity of the beam section is obtained when the normal tensile strain at the bottom fiber \( \beta = \frac{\varepsilon_{tbot}}{\varepsilon_{cr}} \) reaches the maximum tensile strain \( \alpha = \frac{\varepsilon_{trn}}{\varepsilon_{cr}} \). However, the simplified equations for moment capacity [Eqs. (7.39) - (7.42)] are applicable for the compressive stress in elastic region below the compressive yield limit \( \sigma_{cy} = 0.85f'_c \). The check can be made by computing the normalized compressive strain developed at the top fiber \( \lambda_{dv} \) and comparing it to the normalized compressive yield strain \( \omega \).

\[
\lambda_{dv} = \frac{k}{1-k} \beta \leq \omega
\]  

(7.43)

If the compressive strain exceeds the yield limit, the tensile strain must be decreased to the value less than the maximum tensile strain at peak stress until the corresponding compressive strain developed in the section is within the elastic limit.

**7.3.3. Model Comparisons**

Typical ranges of material properties for GFRC presented in Table 7.2 were used in the model comparisons between the full models (Fig. 5.11) and the simplified models (Fig. 7.5). Seven independent material parameters \( (f'\_c, E, \sigma_{cr}, \sigma_{peak}, \varepsilon_{cr}, \sigma_p \text{ and } E_c) \) are varied for three levels: min, mid and max and six parameters \( (\varepsilon_{tcr}, \varepsilon_{tus}, \sigma_{cy}, \varepsilon_{cu} \text{ and } \varepsilon_{cy}) \) are either fixed to a typical value or dependent on the first seven parameters. The total
combinations of the material models generated for the simulations are \(3^7 = 2187\) and graphically presented in Fig. 7.6.

In the full models, the normalized tensile strain \(\beta\) is incrementally imposed from zero to the normalized ultimate tensile strain value \(\beta_{tu} = \varepsilon_{tu}/\varepsilon_{cr}\) and the normalized moment \(M'\) is calculated by the equations in Table 5.10. The maximum value of \(M'\) is chosen as the ultimate moment capacity. On the contrary, the simplified models use only the normalized tensile strain at peak tensile stress \(\beta = \alpha\) in Eqs. (7.40) and (7.42) to calculate the moment capacity. When using \(\beta = \alpha\) causes compressive strain exceeding the compressive yield strain \(\omega\), iterative computations were performed for \(\beta < \alpha\) until the strain does not exceed the yield limit.

Fig. 7.7 compares the normalized ultimate moment capacity predicted by the full and simplified models according to the ranges of material parameters used in Table 7.2. Fig. 7.7(a) presents the first case that \(\beta = \alpha\), the predicted ultimate moment ratio \(M'_{\text{simplified}}/M'_{\text{full}}\) has an average value of 1.0038 and standard deviation of 0.0189. The minimum and maximum ratios are 0.930 and 1.047 respectively. It can be seen that the model predictability of the simplified Eqs. (7.40) and (7.42) are very satisfactory even ignore the possibility of compressive stress exceeding the yield limit or reaching the compressive failure. This implies that typical range of material properties used for GFRC is a tension dominated. Fig. 7.7(b) presents another case that \(\lambda_{dv}\) is not allowed to exceed \(\omega\), which is done by iteratively trialing the value \(\beta\) less than \(\alpha\) such that the compressive strain \(\lambda_{dv}\) decreases to \(\omega\) in the elastic limit (stage 2.1). In this case, it can be seen that the
$M'_{\text{simplified}}$ is generally less than $M'_{\text{full}}$. The predicted ultimate moment ratio $M'_{\text{simplified}}/M'_{\text{full}}$ has an average value of 0.982 and standard deviation of 0.0547. The minimum and maximum ratios are 0.703 and 1.047 respectively. This case provides a conservative moment capacity and it will be used in the design guide for strain-hardening GFRC.

For convenience in estimation of moment capacity and compressive stress developed at the top fiber, the design charts $M_n' - \beta$ and $\lambda_{dv} - \beta$ for range of normalized postcrack tensile modulus $\eta$ according to Eqs. (7.40), (7.42) and (7.43) are presented in Fig. 7.8. For GFRC selected, the value of $\eta$ is known, users can try the maximum tensile strain $\beta = \alpha$ and gets $M_n'$ from $M_n' - \beta$ and $\lambda_{dv}$ form $\lambda_{dv} - \beta$ chart. In the case that $\lambda_{dv}$ exceeds $\omega$, the $\lambda_{dv}$ is set to $\omega$ and backward identify $\beta$ from $\lambda_{dv} - \beta$ and $M_n'$ from $M_n' - \beta$ chart. Once $M_n'$ is known the nominal moment capacity $M_n$ is obtained by the product of $M_n'$ and $M_{cr}$.

7.3.4. Design Examples

7.3.4.1. Example 1: Trough Cover

Design a one-foot-span trough cover to be used in an industrial pavement area as shown in Fig. 7.9(a). The cover is subjected to a uniformly distributed tire pressure of 80 psi. The material used is GFRC with the ultimate compressive strength $f_{c'}$ of 9500 psi, Young’s modulus $E$ of $2.15 \times 10^6$ psi, cracking tensile strength $\sigma_{cr}$ of 850 psi, ultimate tensile strength $\sigma_{peak}$ of 1300 psi, tensile strain at ultimate tensile strength $\varepsilon_{t_{rn}}$ of 0.009 and density of 130 pcf.
Self weight of the cover is negligible compared to the load from a truck wheel; but it is computed here to illustrate the calculations of factored loading. Estimate the self weight of GFRC cover from the design experience that thickness is between 4” and 6”. Assumes the thickness is 6” in the calculation for the self weight.

\[ w_{sw} = 6 \times \frac{130}{12 \times 12 \times 12} = 0.45 \text{ psi} \]

Calculate the ultimate factored load

\[ w_u = 1.2(DL) + 1.6(LL) = 1.2(0.45) + 1.6(80) = 129 \text{ psi} \]

Consider the cover for 1 inch strip and calculate the maximum moment at mid span of the beam

\[ M_u = \frac{w_u L^2}{8} = \frac{129 \times 12^2}{8} = 2,322 \text{ lb-in/in} \]

Next is to find the normalized moment capacity of the GFRC used in the design.

Calculate material parameters according to Eqs. (7.34) – (7.37) as follows:

Cracking tensile strain, \( \varepsilon_{cr} = \frac{\sigma_{cr}}{E} = \frac{850}{2.15 \times 10^6} = 3.953 \times 10^{-4} \)

Normalized tensile strain at peak stress, \( \alpha = \frac{\varepsilon_{trn}}{\varepsilon_{cr}} = \frac{0.009}{3.953 \times 10^{-4}} = 22.8 \)

Normalized postcrack tensile modulus,

\[ \eta = \frac{E_{cr}}{E} = \frac{1300 - 850}{(0.009 - 0.0003953) \times 2.15 \times 10^6} = 0.0243 \]

Normalized compressive yield strain, \( \omega = \frac{\sigma_{cy}}{\sigma_{cr}} = \frac{0.85 \times f'_{c}}{\sigma_{cr}} = \frac{0.85 \times 9500}{850} = 9.5 \)
The normalized moment capacity, neutral axis and compressive strain developed at top fiber can be calculated by Eqs. (7.40), (7.42) and (7.43).

Calculate a constant parameter 1, \( C_1 = \eta \left( \beta^2 - 2\beta + 1 \right) + 2\beta - 1 \)

\[
C_1 = 0.0243 \left( 22.8^2 - 2 \times 22.8 + 1 \right) + 2 \times 22.8 - 1 = 56.15
\]

Neutral axis depth ratio, \( k = \frac{C_1 - \sqrt{\beta^2 C_1}}{C_1 - \beta^2} = \frac{56.15 - \sqrt{22.8^2 \times 56.15}}{56.15 - 22.8} = 0.247 \)

Normalized compressive strain, \( \lambda_{dv} = \frac{k}{1-k} \beta = \frac{0.247}{1-0.247} \times 22.8 = 7.48 < \omega = 9.5 \implies \text{OK} \)

Calculate a constant parameter 2, \( C_2 = C_1 + 2C_1 \beta - \beta^2 \)

\[
C_2 = 56.15 + 2 \times 56.15 \times 22.8 - 22.8^2 = 2097
\]

Normalized moment capacity, \( M'_n = C_2 \frac{k^2 - 2k + 1}{\beta^2} + \frac{2 \beta k^3}{1-k} \)

\[
M'_n = 2097 \frac{0.247^2 - 2 \times 0.247 + 1}{22.8^2} + \frac{2 \times 22.8 \times 0.247^3}{1-0.247} = 3.2
\]

Once the normalized capacity is identified, the cracking moment that satisfies the ultimate moment due to factored load, according to Eqs. (7.38) and (7.39) is calculated.

\[
M_{cr} = \frac{M'_n}{\phi_{cr} M_n} = \frac{M_u}{\phi_{cr} M_n} = \frac{2322}{0.65 \times 3.2} = 1116 \text{ lb-in/in}
\]

Finally, the required thickness is determined by Eq. (7.41)

\[
h = \sqrt[6]{\frac{6M_{cr}}{\sigma_{cr} b}} = \sqrt[6]{\frac{6 \times 1116}{850 \times 1}} = 2.81 \text{ inches}
\]
Let use the thickness of 3.0 inch which is less than the assumed thickness of 6 inches in the estimation of the self weight => (OK).

Alternatively, the design charts presented in Fig. 7.9(b) can be used to quickly estimate the normalized moment capacity. First, draw a vertical line from $\beta = \alpha = 22.8$ to the curve at $\eta = 0.0243$, get $M_n' = 3.3$ from the $M_n' - \beta$ chart and $\lambda_{dv} = 7.7$ from the $\lambda - \beta$ chart. Since $\lambda_{dv}$ is lower than the normalized compressive yield strain $\omega = 9.5$, the obtained $M_n' = 3.3$ is valid. It can be seen that the manually picked values $\lambda_{dv} = 7.7$ and $M_n' = 3.3$ from the design charts are very close to the exact values $\lambda_{dv} = 7.48$ and $M_n' = 3.2$ obtained from the calculation method. Once $M_n'$ is identified, the required cracking moment $M_{cr}$ and thickness $h$ can be calculated as demonstrated before.

7.3.4.2. Example 2: Soil Container

Design a GFRC soil container as shown in Fig. 7.10(a) to be used in the garden. The width, height and length are 20, 30 and 100 inches, respectively. The soil has a moist density of 120 pcf and a friction angle $\theta = 30^\circ$. The estimated weight of plants is 40 psf. Assume that there is a hole at the bottom of the box to drain the water such that water pressure can be ignored in the design. Use GFRC that has compressive strength $f_{c'}$ of 9000 psi, Young’s modulus $E$ of 2.9x10^6 psi, cracking tensile strength $\sigma_{cr}$ of 1000 psi, ultimate tensile strength $\sigma_{peak}$ of 1600 psi and tensile strain at ultimate tensile strength $\epsilon_{trn}$ of 0.012.

To design a soil container, the side wall can be analyzed as a cantilever beam subjected to lateral earth pressure $p_e$ and lateral pressure due to surcharge $p_s$ as shown in Fig. 7.10(b). The net pressure at the bottom panel is zero because the loads that compress
the top face equal to the reaction force acts at the bottom face. However, both ends of the bottom panel are subjected to the moment from side walls; thus, the entire span has a uniform bending moment, equal to the moment at the base of the side wall.

The loads acting to the cantilever beam model can be determined as a retaining wall problem. The active pressure soil coefficient according to the friction angle of the soil is calculated as

\[ C_a = \tan^2 \left( 45 - \frac{\theta}{2} \right) = \tan^2 \left( 45 - \frac{30}{2} \right) = 0.333 \]

The distribution of lateral earth pressure has a triangular shape with the maximum at the bottom. Calculate the lateral earth pressure at 30 inches below the ground level as

\[ p_e = C_a \gamma_s H = 0.333 \times \frac{120}{12 \times 12 \times 12} \times 30 = 0.694 \text{ psi} \]

Conservatively assumes the weight of plant is a surcharge load imposed at the top soil; thus, the uniform lateral pressure due to the surcharge is

\[ p_s = C_a S = 0.333 \times \frac{40}{12 \times 12} = 0.0925 \text{ psi} \]

The bending moment due to triangular and uniformly distributed load is critical at the base level, which can be calculated by static equilibrium

\[ M_u = 1.6 \left( \frac{1}{2} p_e H \frac{H}{3} + p_s H \frac{H}{2} \right) = 1.6 \left( \frac{1}{2} \times 0.694 \times 30 \times \frac{30}{3} + 0.0925 \times 30 \times \frac{30}{2} \right) = 233.2 \text{ lb-in/in} \]

Next is to find the normalized moment capacity of the GFRC used in the design. Calculate material parameters according to Eqs. (7.34) – (7.37) as follows:

Cracking tensile strain, \( \varepsilon_{cr} = \frac{\sigma_{cr}}{E} = \frac{1000}{2.9 \times 10^6} = 3.448 \times 10^{-4} \)
Normalized tensile strain at peak tensile stress, \( \alpha = \frac{\varepsilon_{trn}}{\varepsilon_{cr}} = \frac{0.012}{3.448 \times 10^{-4}} = 34.8 \)

Normalized postcrack tensile modulus,
\[
\eta = \frac{E_{cr}}{E} = \frac{1600 - 1000}{(0.012 - 0.0003448) \times 2.9 \times 10^6} = 0.0178
\]

Normalized compressive yield strain, \( \omega = \frac{\sigma_{cy}}{\sigma_{cr}} = \frac{0.85 \times f'_{c}}{\sigma_{cr}} = \frac{0.85 \times 9000}{1000} = 7.65 \)

The normalized moment capacity, neutral axis and compressive stress developed at the top fiber can be calculated by Eqs. (7.40), (7.42) and (7.43).

Calculate a constant parameter 1, \( C_1 = \eta \left( \beta^2 - 2\beta + 1 \right) + 2\beta - 1 \)
\[
C_1 = 0.0178 \left( 34.8^2 - 2 \times 34.8 + 1 \right) + 2 \times 34.8 - 1 = 88.9
\]

Neutral axis depth ratio, \( k = \frac{C_1 - \sqrt{\beta^2 C_1}}{C_1 - \beta^2} = \frac{88.9 - \sqrt{34.8^2 \times 88.9}}{88.9 - 34.8^2} = 0.213 \)

Normalized compressive strain developed, \( \lambda_{dv} = \frac{k}{1 - k} \beta = \frac{0.213}{1 - 0.213} \times 34.8 = 9.42 > \omega = 7.65 \)

Since the compressive strain developed exceeds the yield limit, the maximum tensile strain \( \beta = \alpha \) can not be used. The value of \( \beta \) less than \( \alpha \) must be trial and error at \( \eta = 0.0178 \) until \( \lambda_{dv} \) does not exceed \( \omega \). This process requires iterative calculations which are presented below.

Try \( \beta = 30 \), then recalculate \( C_1 = 73.9 \), \( k = 0.223 \), \( \lambda = 8.61 > \omega = 7.65 \) => Not OK

Try \( \beta = 25 \), then recalculate \( C_1 = 59.2 \), \( k = 0.235 \), \( \lambda = 7.70 > \omega = 7.65 \) => Not OK
Try $\beta = 24$, then recalculate $C_1 = 56.4$, $k = 0.238$, $\lambda = 7.51 < \omega = 7.65 \Rightarrow$ OK but not economical

Try $\beta = 24.5$, then recalculate $C_1 = 57.8$, $k = 0.237$, $\lambda = 7.60 \approx \omega = 7.65 \Rightarrow$ OK and economical

Once $\beta$ is identified, calculate constant parameter $2$, $C_2 = C_1 + 2C_1\beta - \beta^2$

$$C_2 = 57.8 + 2 \times 57.8 \times 24.5 - 24.5^2 = 2290$$

Normalized moment capacity, $M_n' = C_2 \frac{k^2 - 2k + 1}{\beta^2} + \frac{2\beta k^3}{1-k}$

$$M_n' = 2290 \frac{0.235^2 - 2 \times 0.235 + 1}{24.5^2} + \frac{2 \times 24.5 \times 0.235^3}{1 - 0.235} = 3.06$$

After the normalized capacity is identified, the cracking moment that satisfies the ultimate moment due to factored load can be calculated according to Eqs. (7.38) and (7.39)

$$M_{cr} = \frac{M_n}{M_n'} = \frac{M_u}{\phi_{r3}M_n'} = \frac{233.2}{0.65 \times 3.06} = 117.2 \text{ lb-in/in}$$

Finally, the required thickness is determined by Eq. (7.41)

$$h = \sqrt{\frac{6M_{cr}}{\sigma_{cr} b}} = \sqrt{\frac{6 \times 117.2}{1000 \times 1}} = 0.84 \text{ inches, lets use 1.0 inch.}$$

Since the side wall and the bottom panels are subjected to the same magnitude of moment, the thickness of both panels is the same at 1.0 inch.

The design charts can be used in this case to avoid the lengthy iterative calculations. In Fig. 7.10(c), draw a vertical line from $\beta = \alpha = 34.8$ to the curve at $\eta = 0.0178$, get $M_n' = \cdots$
3.3 from the $M_n' - \beta$ chart and $\lambda_{dv} = 9.6$ from the $\lambda_{dv} - \beta$ chart. Because the compressive strain developed $\lambda_{dv}$ exceeds the limit $\omega$ of 7.65, set the compressive strain to the yield limit ($\lambda_{dv} = \omega = 7.65$). Then draw a horizontal line back to the right meeting the curve at $\eta = 0.0178$, get $\beta = 26$ from the $\lambda_{dv} - \beta$ chart and $M_n' = 3.0$ from the $M_n' - \beta$ chart. These values $\beta = 26$ and $M' = 3.0$ obtained from the design charts are very close to the values $\beta = 24.5$ and $M' = 3.06$ obtained by iterative calculations. Finally, the required cracking moment and thickness can be identified as shown before.

**7.3.5. Discussions**

The design guide is provided for strain-hardening GFRC only where typical material properties are well defined by the manufacture. The unique characteristics of GFRC that has relatively flat postcrack modulus and large tensile strain at peak strength make the approximate bilinear tension and elastic compression model well predict the flexural strength. Other strain-hardening materials such as SIMCON and ECC may require numerical investigations of the model predictability as shown in Fig. 7.7(a and b) before using the simplified Eqs. (7.40) - (7.43). Other uniaxially reinforced composites such as ferrocement and fabric reinforced cement may require additional investigation of failure mode. It is possible that heavily reinforced by several layers of fabrics can cause specimen to fail by shear delimitation instead of flexural failure. In this case, shear delimitation criterion is needed to be developed or shear failure must be prevented.

From design examples 1 and 2, ones can notice that the nominal moment capacity is around 3.0 – 3.2 times the cracking moment. The value significantly greater than 1 implies that the ultimate moment capacity of material is achieved when the tensile strain
is stretched far beyond the cracking level. The reduction factor of 0.65 and loading factors between 1.2 – 1.6 together provide the safety factors around 1.85 – 2.46. Therefore, the designed members under service load condition are expected to carry the moment around 1.2 - 1.7 of the cracking moment. In the case of soil container that the loads are primarily static, the designed members may crack but still function, carrying the load for their service life time. On the contrary, the trough cover that the load is primarily a repeated load. Fatigue due to stress beyond cracking limit may be an issue and it requires future research in this area to amend the provision for the safe design.

7.4. Chapter Conclusions

This chapter proposes a design guideline for strain-softening and strain-hardening fiber reinforced concrete. The guideline for strain-softening is well suite for the SFRC due to some critical values and empirical equations are borrowed from the model developed for SFRC. Other FRC that posses similar material characteristics as SFRC and demonstrate comparable tensile performance can follow this guideline with engineering judgments. The guideline for strain-hardening is applicable for GFRC only since it uses the material characteristics of GFRC that shows relatively flat postcrack modulus with large tensile strain capacity in the simplification from the full analytical model to the simplified design equations.

In strain-softening FRC, simplified analytical equations are used to estimate flexural strength and empirical equations are used to estimate shear capacity. Size effect of a beam deeper than 5 inches is also taken into account by decreasing the postpeak tensile strength with a size-dependent safety factor. Reduction factors for FRC and hybrid FRC-
rebar were suggested as 0.70 and 0.90, respectively base on statistical analysis of small SFRC beam samples. These two factors will be adjusted when comprehensive beam test data, covering various design variables, become available. The guidelines also propose the minimum postpeak tensile strength for flexural members designed with FRC and the minimum flexural reinforcement ratio for hybrid flexural members designed with FRC and steel reinforcement. For serviceability criterion, additional minimum postpeak tensile strength for controlling crack width due to shrinkage and temperature is proposed for the FRC systems.

The guidelines for strain-softening are best suited for thin structural applications such as slabs and footing systems. For these thin members, the size effect is minimal and the full section can be efficiently utilized. Shear stress relative to its strength is generally not critical and shear reinforcement is not required. Three design examples, slab on grade, four-span floor slab and wall footing, are presented to illustrate the calculation steps. Finally, the it demonstrates how to use the Excel worksheet to back calculate postpeak tensile strength from load-deflection response obtained from the Average Residual Strength (ARS) test method ASTM C 1399.

In strain-hardening GFRC, the simplified analytical equations are used to estimate flexural strength. No guideline for shear, size effect and minimum reinforcement. According to the design examples 1 and 2, it is found that the normalized moment capacity is between 3.0 and 3.2 times the cracking moment. When consider the total safety factors around 1.8 – 2.5 used in the design process, the structural members will carry the moment at service load around 1.2 – 1.7 times the cracking moment. If the
members are to be used in carrying repeated loads such as trough cover in Example 1, fatigue failure may be a problem as the strain at service load exceeds the cracking limit. Future research in fatigue is required for using GFRC in repeated load applications. The method presented here is better used for the applications in static mode such as soil container that the panel may crack but not fail. The crack width of strain-hardening material is expected to be very small and micro crack will not cause problems for this application.

**Notation**

*The following symbols are used in this paper*

- $A_s$ = area of steel reinforcement;
- $a$ = depth of stress block;
- $b$ = width of rectangular beam;
- $b_w$ = width of rectangular or T-beam web;
- $C_1$ = parameter for calculating neutral axis depth ratio of GFRC;
- $C_2$ = parameter for calculating normalized nominal moment capacity of GFRC;
- $C_a$ = active pressure soil coefficient;
- $C_m$ = ACI moment coefficient;
- $C_v$ = ACI shear coefficient;
- $DL$ = dead load;
- $d$ = depth of reinforcement or effective depth;
- $E$ = tensile modulus of FRC;
$E_c$ = compressive modulus of FRC;
$E_{cr}$ = postcrack tensile modulus;
$E_s$ = Young’s modulus of steel;
$f_c'$ = uniaxial cylinder compressive strength of concrete;
$f_{cm}$ = mean uniaxial cylinder compressive strength of concrete;
$f_{sy}$ = steel yield strength;
$G_1, G_2$ = parameters for calculating minimum flexural reinforcement ratio;
$H$ = height of retaining wall;
$h$ = full height of beam;
$k$ = neutral axis depth ratio;
$L$ = clear span;
$LL$ = live load;
$M$ = moment;
$M'_{full}$ = normalized moment capacity predicted by the full model;
$M_n$ = nominal moment capacity;
$M'_{simplified}$ = normalized moment capacity predicted by the simplified model;
$M_u$ = ultimate moment capacity;
$n$ = modulus ratio between steel and FRC;
$p_e$ = lateral earth pressure;
$p_s$ = lateral pressure due to the surcharge;
$q_a$ = allowable soil pressure;
$q_n$ = net allowable soil pressure;
\( q_{nu} \) = net factored soil pressure;
\( S \) = surcharge pressure at top soil;
\( s \) = standard deviation;
\( V_c \) = shear capacity of concrete section;
\( V_{FRC} \) = shear capacity of FRC section;
\( V_f \) = fiber volume fraction;
\( V_{SFRC} \) = shear capacity of SFRC section;
\( V_u \) = ultimate shear load;
\( w_{sw} \) = distributed self weight;
\( w_u \) = factored distributed load;
\( X \) = shear span;
\( \alpha \) = normalized depth of reinforcement \((d/h)\);
\( \beta \) = normalized tensile strain at the bottom fiber \((\varepsilon_{bot}/\varepsilon_{cr})\);
\( \beta_{nu} \) = normalized ultimate tensile strain \((\varepsilon_{u}/\varepsilon_{cr})\);
\( \varepsilon \) = strain;
\( \varepsilon_{bot} \) = tensile strain of FRC at the bottom fiber;
\( \varepsilon_{trn} \) = transition tensile strain (or strain at peak) of FRC;
\( \phi_b \) = reduction factor for bending in reinforced concrete beams;
\( \phi_s \) = reduction factor for shear in reinforced concrete beams;
\( \phi_{r1}, \phi_{r2} \) = reduction factor for bending in strain-softening FRC beams and hybrid beams reinforcement, respectively;
\( \phi_3 \) = reduction factor for bending in strain-hardening GFRC beams;

\( \gamma \) = normalized compressive modulus of FRC \((E_c/E)\);

\( \gamma_s \) = soil density;

\( \eta \) = normalized postcrack tensile modulus of FRC \((E_{cr}/E)\);

\( \kappa \) = normalized steel yield strain \((\varepsilon_{sy}/\varepsilon_{cr})\);

\( \kappa_h \) = size dependent safety factor;

\( \lambda_{cu} \) = normalized ultimate compressive strain of FRC \((\varepsilon_{cu}/\varepsilon_{cr})\);

\( \lambda_{dv} \) = normalized compressive strain developed at the top fiber of FRC \((\varepsilon_{ctop}/\varepsilon_{cr})\);

\( \mu \) = normalized postpeak tensile strength of FRC \((\sigma_p/\sigma_{cr})\);

\( \mu_{crit} \) = critical normalized postpeak tensile strength that changes deflection-softening to deflection-hardening response;

\( \mu_{min,flex} \) = minimum normalized postpeak tensile strength for flexure;

\( \mu_{min,shrk} \) = minimum normalized postpeak tensile strength for shrinkage and temperature;

\( \theta \) = friction angle of soil;

\( \rho_g \) = reinforcement ratio per gross area;

\( \rho_{g,bal} \) = reinforcement ratio per gross area at balance failure;

\( \rho_{g,min,flex} \) = minimum flexural reinforcement ratio per gross area;

\( \rho_{g,min,shrk} \) = minimum reinforcement ratio per gross area for shrinkage and temperature;
\( \sigma \) = stress;
\( \sigma_p \) = postpeak tensile strength of FRC;
\( \sigma_{peak} \) = peak or ultimate tensile strength of FRC;
\( \sigma_{pm} \) = mean postpeak tensile strength of FRC from bending test; and
\( \omega \) = normalized compressive yield strain of FRC \( (\varepsilon_{cy}/\varepsilon_{cr}) \).

**Super Script**

\( ( ') \) = normalized with either moment or curvature at cracking.

**Sub Script**

\( cr \) = at cracking tensile strain of FRC;
\( cy \) = at compressive yield strain of FRC;
\( sy \) = at steel yield strain;
\( tu \) = at ultimate tensile strain of FRC; and
\( cu \) = at ultimate compressive strain of FRC;
Table 7.1. Inverse Analysis Results of Load-Deflection Response of SFRC Mix

<table>
<thead>
<tr>
<th>Sample</th>
<th>Post crack tensile strength (psi)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>231</td>
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<tr>
<td>2</td>
<td>265</td>
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<tr>
<td>3</td>
<td>191</td>
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<tr>
<td>Average</td>
<td>229</td>
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<tr>
<td>Standard deviation</td>
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</tr>
<tr>
<td>Material properties</td>
<td>Symbols</td>
</tr>
<tr>
<td>----------------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>Ultimate Compressive strength (psi)</td>
<td>$f'_c$</td>
</tr>
<tr>
<td>Tensile Young's modulus (psi)</td>
<td>$E$</td>
</tr>
<tr>
<td>Cracking tensile strength (psi)</td>
<td>$\sigma_{cr}$</td>
</tr>
<tr>
<td>Cracking tensile strain (in/in)</td>
<td>$\varepsilon_{cr}$</td>
</tr>
<tr>
<td>Ultimate tensile strength (psi)</td>
<td>$\sigma_{peak}$</td>
</tr>
<tr>
<td>Tensile strain at ultimate tensile strength (in/in)</td>
<td>$\varepsilon_{trn}$</td>
</tr>
<tr>
<td>Postpeak tensile strength (psi)</td>
<td>$\sigma_p$</td>
</tr>
<tr>
<td>Ultimate tensile strain (in/in)</td>
<td>$\varepsilon_{tu}$</td>
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<tr>
<td>Compressive modulus (psi)</td>
<td>$E_c$</td>
</tr>
<tr>
<td>Plastic compressive stress (psi)</td>
<td>$\sigma_{cy}$</td>
</tr>
<tr>
<td>Ultimate compressive strain (in/in)</td>
<td>$\varepsilon_{cu}$</td>
</tr>
<tr>
<td>Plastic compressive strain (in/in)</td>
<td>$\varepsilon_{cy}$</td>
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</tbody>
</table>
Fig. 7.1. Material model for single reinforced concrete design: (a) compression model; (b) tension model; (c) steel model; (d) beam cross section
Fig. 7.2. Four-span floor slab
Fig. 7.3. Wall footing
Beam Size

<table>
<thead>
<tr>
<th>Method</th>
<th>Value</th>
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<td>0</td>
</tr>
<tr>
<td>b =</td>
<td>4.03</td>
</tr>
<tr>
<td>h =</td>
<td>4.06</td>
</tr>
<tr>
<td>L =</td>
<td>12</td>
</tr>
<tr>
<td>S =</td>
<td>4</td>
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Material Model

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<tr>
<td>$\varepsilon_{cr} = 0.0001316$</td>
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</tr>
<tr>
<td>strain</td>
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<td>stress</td>
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<table>
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<td>$\sigma_{cr} = 530.33$</td>
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<td>$\beta_{tu} = 190.0$</td>
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<table>
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<td>$\sigma_{cy} = 4252.65$</td>
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<tr>
<td>$\lambda_{cu} = 26.60$</td>
<td>$\epsilon_{cu} = 0.0035000$</td>
<td>$\sigma_{cu} = 4252.65$</td>
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</tbody>
</table>

![Load Deflection Response](image)

- Experiment
- Average
- Model Prediction

<table>
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<th>Load, P</th>
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</tr>
<tr>
<td>9,000</td>
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Fig. 7.4. Inverse analysis of SFRC using Excel worksheet: (a) input data; (b) matching predicted load-deflection response to experimental post crack response of the average residual strength test method; (c)&(d) tension and compression models obtained from inverse analysis.
Fig. 7.5. Simplified material model for strain-hardening: (a) tension model; (b) compression model
Fig. 7.6. Range of material models generated: (a) tension model; (b) compression model
Fig. 7.7. Compare normalized ultimate moment capacity predicted by simplified model and full model: (a) elastic compression without yield limit; (b) elastic compression with limit at yielding
Fig. 7.8. Normalized design chart for GFRC
Fig. 7.9. Trough and cover in the industrial pavement: (a) geometry; (b) design chart
Fig. 7.10. Soil container: (a) dimension; (b) analysis model; (c) design chart
CHAPTER 8
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

8.1. Executive Summary and Conclusions
Multi-scale modeling of fiber and fabric reinforced concrete has been developed, starting from the micromechanics level of yarn pullout mechanism at yarn-matrix interface. This mechanism can be used to simulate yarn bridging force at crack locations in tension specimen in the meso scale level to obtain a characteristic tensile stress strain response of material. Finally the tensile response is used to formulate moment-curvature relationship which can be used in conjunction with moment area method to predict flexural behaviors of a beam at the macro structural level.

In order to develop analytical tools for fiber reinforced concrete (FRC) primarily used in flexural mode, a moment-curvature relationship linking uniaxial compressive and tensile responses to the overall flexural capacity is selected. With reasonable assumptions, the analytical equations for moment-curvature relationship can be simplified to obtain sets of design equations. In addition, they are combined with other peripheral information such as the estimation of typical material properties gathered from related literatures to form a draft of design guide for FRC.

8.2. Summary of Fiber/Fabric Pullout Model and Recommendations
The first task of the research program started with the development of yarn pullout model. The existing fiber pullout model developed by Naaman (1991a, b) was reformulated in order to fit the apparatus setup used in the fabric pullout tests. It is notable that when fabric is embedded in the matrix, the pullout-slip response is generally higher than that of the yarn embedded in the same matrix due to mechanical
anchorage of the transverse yarn of the fabric. Generally speaking, the pullout characteristic of the fabric and the yarn appear to be the same in the prepeak region, but the fabric has higher pullout capacity. In the postpeak some fabric-matrix composites show staggering drop in strength while the yarns show a smooth drop. The newly developed yarn pullout model was used in characterization of bond strength parameters of the fabric pullout tests, varying in fabric types, mixture compositions, processing methods, and embedded lengths. It was found that the bond strength was strongly depending on the fabric types, mixture compositions and processing methods and less affected by the embedded lengths. The AR-glass fabric demonstrated the best bond strength when using with flyash mix. Due to the nature of straight yarn in both orthogonal directions and large fabric opening, the cement paste could fill up the void and then led to dense interface zone. On the contrary, PVA fabric with cast method showed the worst bond strength; the yarn was loosely bundled by several filaments and the fabric opening was very tight. Cement paste had poor penetration to the fabric openings and the interspatial of the filaments and this led to poor bonding properties.

For better characterization of bond properties, a nonlinear finite difference fabric pullout model was developed in order to address three main aspects of fabric-cement matrix interaction: nonlinear bond slip relation at the interface, tensile stress strain behavior of the longitudinal yarn that may begin with initial slack, and nonlinear spring model simulating transverse yarn at junctions. The model assumes the axial stiffness of the longitudinal yarn is much less than the axial stiffness of the matrix so that the matrix strain can be ignored in the pullout mechanism. This assumption is reasonable for most
composites used as construction materials that normally contain small amount of reinforcement to minimize cost but may not be true for the composites used in aerospace industries in which the composites are heavily reinforced with fibers to minimize weight. This newly developed tool was used in the parametric study and the results revealed that the shape of bond-slip model closely resembled the pullout-slip response; the effect of slack in longitudinal yarn caused the pullout response to be more flexible and slightly dropped in maximum strength; the resistant provided by transverse yarn showed similar effect to the increase in bond strength that it increased the pullout strength. The same set of AR-glass fabric reinforced cement characterized by the yarn pullout model to obtain equivalent bond properties was reanalyzed by the finite difference fabric pullout model in order to identify the contribution between the bond along the longitudinal yarn and the resistance due to anchorage of the transverse yarns. It was found that transverse yarns offered tangible resistance to the pullout strength, especially when slip was small relative to the junction size. At a higher slip the junction failed and transverse yarn could not contribute to the pullout strength causing a staggering drop in strength as observed in the test. Thus, the bond at longitudinal yarn-matrix interface was the remaining load transfer mechanism. The applicability of the fabric pullout model is not limited to only the fabric composites, but it can also be used to characterize bond properties of other cement based composites such as steel reinforcement embedded in concrete. In this case, the slack in longitudinal reinforcement and spring needs to be turn off.

The future development in this area is to refine the finite difference fabric pullout model by taking the matrix strain into account; this will make the model capable of
modeling composite containing high fiber contents. On the experimental side, the current model can be used to characterize new types of reinforcing materials to be used with concrete such as high grade steel and fiber reinforced plastic (FRP). The advantages of using this tool in characterizations are two folds: computation cost is much less expensive than using nonlinear finite element software package and the calibrated bond parameters of various composite systems can be directly compared.

8.3. Summary of Tension Stiffening Model and Recommendations

The nonlinear finite difference fabric pullout model was originally developed for one side pullout problem under displacement control. The model was extended to cover both one side and two side pullout problems so that the model can be used to simulate yarn bridging force at crack locations at the end and in the middle of a tension specimen. In tension stiffening model, the load control is used instead of displacement control to provide an efficient algorithm for numerical simulations. Each pullout segment is subject to the same tension load at crack location and it can be solved independently. Using displacement control in this case leads to non unique solution that the loads at different crack locations are different from one another and slip at each crack location can be either snap back or continue to increase as the load decreases. Matrix strength is used as a cracking criterion in which the crack is detected when tensile stress exceeds local matrix strength. The matrix used in the current model is assumed to be brittle and has no postpeak capacity. In simulation of crack evolution, two crack patterns: deterministic crack and stochastic crack can be achieved by manipulating matrix strength distribution along the tension specimen. By slightly lowering matrix strength in a symmetrical
manner the deterministic crack pattern will appear in a predictable manner; i.e., first crack at the middle, second at the edge of the end grip, then at one-quarter, then at one eight so on and so forth. On the other hand, the stochastic crack pattern can be achieved by randomly generating matrix strength distribution. When the local tensile stress exceeds, the matrix strength crack will appear randomly.

The model was used in the parametric study and the results showed that the increase in bond strength proportionally increased the postcrack tensile stiffness of the tension member. Likewise, the increase in spring strength that simulated restraining effect of the traverse yarn showed the improvement in postcrack stiffness. The increase in matrix strength enhanced the postcrack stiffness as well as the first cracking strength. On the other hand, the slack in longitudinal yarn causes large tensile strain in the early crack evolution but it still yielded relatively the same postcrack stiffness in the late tensile response. All simulations of tension members failed in a brittle manner due to the assumption of yarn rupture at its ultimate load.

The fabric pullout model was used to explain the tension stiffening in reinforced concrete tension members. In this case the slack and spring options were turn off, only the longitudinal reinforcement and its bonding at the interface were used. The pullout slip response obtained from the pullout test was used in the simulations and the results showed good agreement with the tension test results. In addition, the model was used to explain the tensile behavior of fabric reinforced concrete using the equivalent bond-slip relation obtained from the pullout test. The simulation agreed well with the experimental results for most parts; however, it slightly overshot the experimental ultimate load. This
was due to the model assuming all yarns failed simultaneously at the ultimate yarn strength. If the yarns were assumed to fail in a progressive manner that the weakest one failed first and the load was redistributed to the remaining yarns, causing successive failure, the model could fit the experimental ultimate load for this case.

Future development of this model will be the implementation of tensile stress crack width relationship for cracking criterion such that the cracked tension specimen can transfer the forces in the yarn as well as across the crack. This will make the model capable of predicting tension stiffening in a hybrid reinforced concrete the uses both continuous reinforcement and discrete fibers in strengthen concrete matrix.

8.4. Summary of Flexural Modeling and Recommendations

Macroscopic behaviors of reinforced concrete flexural members are primarily governed by the weaker tensile response than the compressive response. Two generic uniaxial constitutive laws, each of which consists of tension and compression models, were used in derivation of moment-curvature relationship and proposed for predicting flexural behaviors of two classes of FRC: strain-softening and strain-hardening. The first model is applicable for strain-softening FRC only while the second one is capable of modeling of both strain-softening and strain-hardening FRC. The closed form solution for moment-curvature relationship of the first model is much simpler than the second model due to fewer material parameters used in the derivation. However, the more comprehensive closed form solution of the second model can capture the more realistic strain-softening behavior. For strain-hardening FRC, the second model is the only choice in modeling.
Finally, the closed form solutions of each model were simplified to a single design equation for predicting moment capacity of a beam.

The parametric study of the strain-softening model revealed that the moment capacity largely depended on the postpeak tensile strength and was less sensitive to the compressive strength as long as the compressive strength was several times higher than the tensile strength. The critical level of postpeak tensile strength that shifts flexural response from deflection-softening to deflection-hardening was derived explicitly and the critical value was identified to be 0.35 for typical FRC. Another parametric study was conducted on stain-hardening FRC using the strain-softening/hardening model. The results revealed that the postcrack tensile modulus and tensile strain capacity at the peak strength were the most sensitive factors to the flexural capacity of the beam. Finally, both models were used in the simulations of FRC beams and the results revealed that the direct use of uniaxial tensile response tended to underpredict the flexural response. However, it was observed that the model prediction improved when material had higher postpeak tensile capacity. This may be due to the fact that the discrete fibers used to increase the postpeak capacity can suppress or delay crack initiation from the weakest point in material. Consequently, the random flaw distribution has negligible effect to the failure mechanism of composite systems.

Future research in this area is to establish material database for each particular composite system such as SFRC and GFRC. Material parameters such as Young’s modulus, cracking strength, and postpeak tensile strength can be empirically related to input variables such as fiber types, slenderness ratio, and volume. This will provide a
platform for concrete producers to quantify their materials to standardized material
dparameters and engineers can select proper material properties for their designs.

8.5. Summary of Hybrid Flexural Modeling and Recommendations

The concept of using moment-curvature relationship to predict flexural behavior of FRC
was extended to the case of hybrid system that uses both conventional steel reinforcement
and FRC. It is generally accepted that the continuous reinforcement is more efficient than
the discrete fiber in terms of flexural capacity. However, laying down reinforcement is
more labor-intensive than pouring FRC at the job sites. The closed form solution for
moment-curvature response of the hybrid system was derived and it is applicable for
three commonly used composite systems: conventional reinforced concrete, strain-
softening FRC, and hybrid reinforced concrete (HRC). The design chart representing
normalized moment capacity as a function of reinforcement ratio and normalized
postpeak tensile strength was proposed. The chart provides a convenient tool for selecting
a combination of the two reinforcing mechanisms to achieve the required moment
capacity.

The proposed model was compared to the RILEM model and the experiment results of
the two beam series: SFRC and hybrid SFRC-steel rebar. In SFRC beams, the RILEM
predictions were slightly higher than the experimental results while the proposed model
predictions were slightly less. The predictions of the two models showed discrepancy for
beams having low fiber contents but their predictions became close to one another for
beams having higher fiber contents. This was due to the flexural behavior of SFRC
beams was mainly controlled by the tension models used in both methods. At low fiber
contents the postpeak tensile response of the two tension models was somewhat different and the difference became less and less when more fibers were added to increase the postpeak tensile strength. In the hybrid system, both RILEM and the proposed model showed improvement in model predictions compared to the experimental results. In addition, their simulations were close to one another. This could be explained by the flexural behavior of the hybrid beam which was more controlled by the steel reinforcement, and the same steel model was used in both methods. Finally, the simplified equation for predicting moment capacity was evaluated with test results. The prediction to experiment ratio was statistically analyzed to identify the reduction factors, which was 0.74 for the SFRC system and 0.862 for the hybrid system.

The minimum flexural reinforcement ratio for the hybrid system was also proposed using the same concept as ACI-318 (2005) that the minimum flexural reinforcement must be provided so that the moment capacity of the reinforced beam after crack would be at least equal to the cracking moment capacity of the unreinforced section. The proposed minimum reinforcement ratio was compared to other models: ACI (2005), Shehata et al. (2003), RILEM proposed by Vandewalle (2003) and experimental results from Bosco et al. (1989, 1990). For the conventional reinforced concrete system, the comparison showed that ACI was the most conservative while the proposed model estimated the required minimum reinforcement in between the more conservative Shehata’s and the less conservative RILEM; the model agreed well with the experimental results. Out of the four models used in the comparison, only the proposed and RILEM models can utilize the benefit of fibers in concrete mix. Both allow minimum reinforcement to be lower
when concrete has higher postpeak tensile capacity due to fibers in the mix. Without experimental data on minimum reinforcement for hybrid reinforced concrete, it cannot be justified which model suggest a better value.

Hybrid reinforced concrete offers excellent strength, large ductility, high impact resistance and good crack width control. It is therefore an ideal composite for making precast panels that need light weight for lowering maneuvering cost and high impact resistant to avoid damage in handling of the products. The hybrid composite is also well suited for structures that need tight crack width such as water retaining structures. Premium structures requiring extra ductility to survive earthquake loads or needing high fatigue resistance to endure repeated loads can be built with this hybrid composite. Future research in this area is to include the effect of axial force in addition to the bending moment in the derivation of moment-curvature response. The enhancement will extend the applicability of the model in several types of thin shell structures that are normally subject to both flexural and axial loadings.

8.6. Summary of Design Guidelines and Recommendations

The closed form solutions for moment-curvature relationship derived from three generic material models: strain-softening FRC, strain-softening/hardening FRC, and hybrid FRC-steel rebar models were simplified to obtain design equations for predicting flexural strength of a beam section. Design guidelines were proposed for two separate classes of FRC: strain-softening FRC and strain-hardening GFRC.

The guideline for strain-softening FRC is applicable for FRC beams and HRC beams. The guideline covers flexural strength, shear capacity, minimum flexural reinforcement,
minimum postpeak tensile strength for flexure, and minimum postpeak tensile strength for temperature and shrinkage. It also addresses the size effect that the deeper beams have lower strength than the shallower ones. The size dependent safety factor from the RILEM model is adopted to decrease the postpeak tensile strength of the proposed model. Since Young’s modulus and tensile strength of strain-softening FRC is expected to be slightly greater than those of plain concrete, it is conservatively assumed that the parameters can be estimated from the formulas for plain concrete given by ACI-318. The plastic compressive strength of FRC is assumed to be 0.85fc‘ similar to the level used in the RILEM model. Only the postpeak tensile strength needs to be identified from inverse analysis of load-deflection from bending test. The equation for shear capacity of an FRC section is modified from the ACI shear equation for plain concrete by replacing the effective depth d to the full depth h. Slightly gain in shear capacity by a factor of h/d is a reasonable estimation as the FRC is tougher than plain concrete and its shear capacity is expected to be higher. On the other hand, the shear capacity of the HRC is adopted from the empirical shear equation for SFRC beams containing flexural reinforcement proposed by Parra-Montesinos (2006). Analytical equations for minimum postpeak tensile strength and minimum reinforcement ratio were derived explicitly with an assumption that the ultimate flexural strength is greater than the cracking moment of an unreinforced section with reasonable safety factors, 0.40/0.35 for postpeak tensile strength and 3/2 for reinforcement ratio. In addition, the equation for minimum postpeak tensile strength to control crack width due to temperature and shrinkage was proposed. This minimum level
provides the same tensile capacity of a reinforced concrete section designed by ACI approach.

Future development of the design guide for strain-softening FRC needs a comprehensive beam test data that addresses at least four main variables: beam depths, fiber types and their characteristics, fiber volume fractions and reinforcement ratios to refine the size dependent safety factor, and reduction factors to be used with the proposed design equations. In addition, the minimum reinforcement ratio can be included in the experimental program to verify the validity of the proposed minimum reinforcement ratio for different types of fibers and their volume fractions.

The second set of guideline is applicable for predicting moment capacity of the GFRC members only. With unique characteristics of GFRC that has relatively flat postcrack modulus and large strain at peak tensile strength, the closed form solutions for strain-softening/hardening were simplified to obtain design equations for estimation of flexural capacity. A pair of design charts, normalized moment capacity and compressive strain at top fiber as a function of normalized tensile strain and postcrack modulus, was proposed for using in the design routine. It was found that typical range of GFRC could result in the moment capacity 3.0 – 3.2 times higher than its cracking moment. With the loading and reduction factors used by ACI-318 (2005) yielding safety factor around 1.8 - 2.5, the moment at service load is around 1.2 – 1.7 times the cracking moment. This raises the concern of fatigue failure as materials are subjected to repeated loading above the cracking limit.
Future development of design guide for strain-hardening GFRC should focus on fatigue issue and identify the upper limit how far we can utilize the material beyond cracking level for structures primarily in static mode and those under repeated loadings. The research in this area requires both theoretical development as well as experimental program. Moreover, the aging effect on GFRC that strength can significantly degrade with time should be incorporated in the design.
References


